Testing, Monitoring, and Dating Structural Changes in Exchange Rate Regimes

Achim Zeileis

http://eeecon.uibk.ac.at/~zeileis/

Overview

- Motivation
  - Exchange rate regimes
  - Exchange rate regression
  - What is the new Chinese exchange rate regime?
- Structural change tools
  - Model frame
  - Testing
  - Monitoring
  - Dating
- Application: Indian exchange rate regimes
- Software
- Summary

Exchange rate regimes

FX regime of a country: Determines how currency is managed wrt foreign currencies.

- *Floating:* Currency is allowed to fluctuate based on market forces.
- *Pegged:* Currency has limited flexibility when compared with a basket of currencies or a single currency.
- *Fixed:* Direct convertibility to another currency.

Problem: The *de facto* and *de jure* FX regime in operation in a country often differ.

$\Rightarrow$ Data-driven classification of FX regimes.

Exchange rate regression

FX regime classification: Workhorse is a linear regression model based on log-returns of cross-currency exchange rates (with respect to some floating reference currency).

Of particular interest: China gave up on a fixed exchange rate to the US dollar (USD) on 2005-07-21. The People's Bank of China announced that the Chinese yuan (CNY) would no longer be pegged to the USD but to a basket of currencies with greater flexibility.

Basket: Here, log-returns of USD, JPY, EUR, and GBP (all wrt CHF).

Results: For the first three months (up to 2005-10-31, $n = 68$) a plain USD peg is still in operation.

Exchange rate regression

Results: Ordinary least squares (OLS) estimation gives

$$\begin{aligned}\mathrm{CNY}_i &= \underset{(0.004)}{0.005} + \underset{(0.009)}{0.9997}\,\mathrm{USD}_i + \underset{(0.011)}{0.005}\,\mathrm{JPY}_i \\ &\quad - \underset{(0.027)}{0.014}\,\mathrm{EUR}_i - \underset{(0.015)}{0.008}\,\mathrm{GBP}_i + \widehat{\varepsilon}_i\end{aligned}$$

Only the USD coefficient is significantly different from 0 (but not from 1).

The error standard deviation is tiny with $\widehat{\sigma} = 0.028$ leading to $R^2 = 0.998$.

Exchange rate regression

Questions:

1. Is this model for the period 2005-07-26 to 2005-10-31 stable or is there evidence that China kept changing its FX regime after 2005-07-26? (***testing***)
2. Depending on the answer to the first question:
   - Does the CNY stay pegged to the USD in the future (starting from November 2005? (***monitoring***)
   - When and how did the Chinese FX regime change? (***dating***)

Exchange rate regression

In practice: Rolling regressions are often used to answer these questions by tracking the evolution of the FX regime in operation.

More formally: Structural change techniques can be adapted to the FX regression to estimate and test the stability of FX regimes.

Problem: Unlike many other linear regression models, the stability of the error variance (fluctuation band) is of interest as well.

Solution: Employ an (approximately) normal regression estimated by ML where the variance is a full model parameter.

Model frame

Generic idea: Consider a regression model for n ordered observations $y_i \mid x_i$ with k-dimensional parameter θ.

Objective function: $\Psi(y_i, x_i, \theta)$ for observations $i = 1, \dots, n$.

$$\widehat{\theta} \;=\; \underset{\theta}{\text{argmin}} \sum_{i=1}^{n} \Psi(y_i, x_i, \theta).$$

Score function: Parameter estimates also implicitly defined by score (or estimating) function $\psi(y, x, \theta) = \partial\Psi(y, x, \theta)/\partial\theta$.

$$\sum_{i=1}^{n} \psi(y_i, x_i, \widehat{\theta}) \;=\; 0.$$

Examples: OLS, maximum likelihood (ML), instrumental variables, quasi-ML, robust M-estimation.

Model frame

For the standard linear regression model

$$y_i \; = \; x_i^\top \beta \; + \; \varepsilon_i$$

with coefficients β and error variance σ^2 one can either treat σ^2 as a nuisance parameter $\theta = \beta$ or include it as $\theta = (\beta, \sigma^2)$.

In the former case, the estimating functions are $\psi = \psi_\beta$

$$\psi_\beta(y, x, \beta) \; = \; (y - x^\top \beta) \, x$$

and in the latter case, they have an additional component

$$\psi_{\sigma^2}(y, x, \beta, \sigma^2) \; = \; (y - x^\top \beta)^2 - \sigma^2.$$

and $\psi = (\psi_\beta, \psi_{\sigma^2})$. This is used for FX regressions.

Model frame

Testing: Given that a model with parameter $\widehat{\theta}$ has been estimated for these *n* observations, the question is whether this is appropriate or: *Are the parameters stable or did they change through the sample period* $i = 1, \dots, n$*?*

Monitoring: Given that a stable model could be established for these *n* observations, the question is whether it remains stable in the future or: *Are incoming observations for* $i > n$ *still consistent with the established model or do the parameters change?*

Dating: Given that there is evidence for a structural change in $i = 1, \dots, n$, it might be possible that stable regression relationships can be found on subsets of the data. *How many segments are in the data? Where are the breakpoints?*

Testing

Idea: Estimate model with $\widehat{\theta}$ under null hypothesis of parameter stability

$$H_0: \ \theta_i = \theta_0 \qquad (i = 1, \dots, n)$$

and capture systematic deviations of scores from zero mean in an empirical fluctuation process:

$$efp(t) \ = \ \widehat{B}^{-1/2} \ n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \hat{\psi}(y_i, x_i, \widehat{\theta}) \qquad (0 \le t \le 1).$$

Functional central limit theorem: Under H_0 and regularity assumptions empirical fluctuation process converges to k-dimensional Brownian bridge

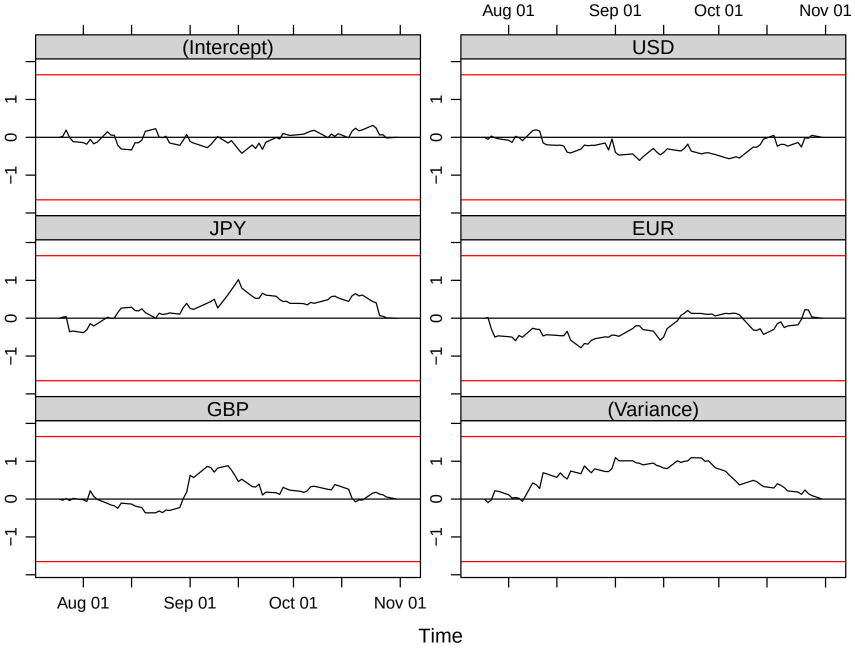
$$efp(\cdot) \ \stackrel{d}{\longrightarrow} \ W^0(\cdot).$$

Testing

Testing procedure:

- Empirical fluctuation processes captures fluctuation in estimating functions.
- Theoretical limiting process is known.
- Choose boundaries which are crossed by the limiting process (or some functional of it) only with a known probability α.
- If the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large $\Rightarrow$ reject the null hypothesis.

Testing



(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug 01
Sep 01
Oct 01
Nov 01
1
0
−1
Time

Testing

More formally: These boundaries correspond to critical values for a double maximum test statistic

$$\max_{j=1,\dots,k} \max_{i=1,\dots,n} |efp_j(i/n)|$$

which is 1.097 for the Chinese FX regression ($p = 0.697$).

Alternatively: Employ other test statistics $\lambda(efp(t))$ for aggregation.

Special cases: This class contains various well-known tests from the statistics and econometrics literature, e.g., Andrews' sup*LM* test, Nyblom-Hansen test, OLS-based CUSUM/MOSUM tests.

Testing

Nyblom-Hansen test: The test was designed for a random-walk alternative and employs a Cramér-von Mises functional.

$$\frac{1}{n}\sum_{i=1}^{n}\left|\left|efp\left(\frac{i}{n}\right)\right|\right|_2^2.$$

For CNY regression: 1.012 ($p = 0.364$).

Andrews' sup*LM* test: This test is designed for a single shift alternative (with unknown timing) and employs the supremum of *LM* statistics for this alternative.

$$\sup_{t\in\Pi} LM(t) \;=\; \sup_{t\in\Pi}\frac{||efp(t)||_2^2}{t\,(1-t)}.$$

For CNY regression: 10.055 ($p = 0.766$), using $\Pi = [0.1, 0.9]$.

Monitoring

Idea: Fluctuation tests can be applied sequentially to monitor regression models.

More formally: Sequentially test the null hypothesis

$$H_0: \theta_i = \theta_0 \qquad (i > n)$$

against the alternative that θ_i changes at some time in the future $i > n$ (corresponding to $t > 1$).

Basic assumption: The model parameters are stable $\theta_i = \theta_0$ in the history period $i = 1, \dots, n$ ($0 \le t \le 1$).

Monitoring

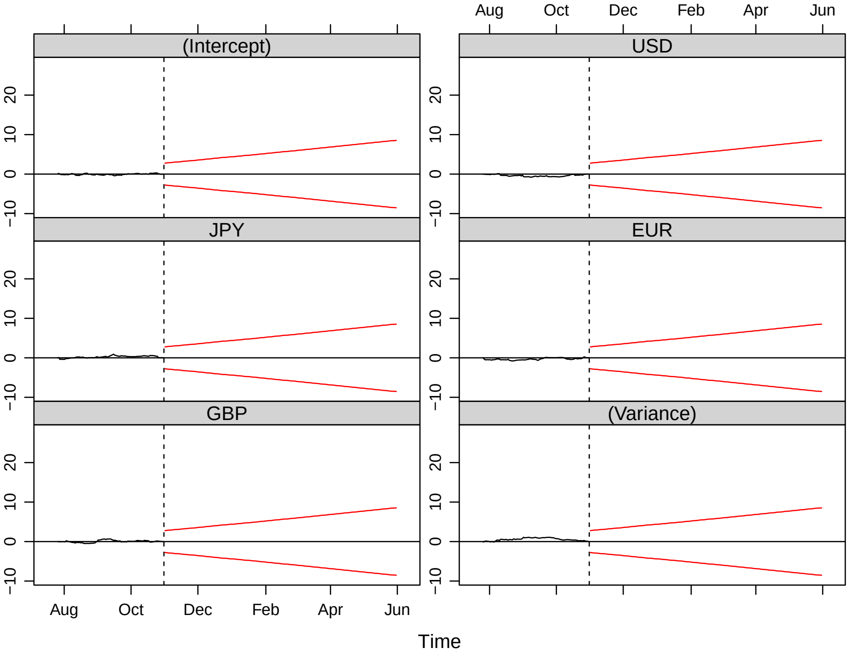
Test statistics: Update $efp(t)$, and re-compute $\lambda(efp(t))$ in the monitoring period $1 \le t \le T$.

Critical values: For sequential testing not only a single critical value is needed, but a full boundary function $b(t)$ that satisfies

$$1 - \alpha \; = \; \mathrm{P}(\lambda(W^0(t)) \le b(t) \mid t \in [1, T])$$

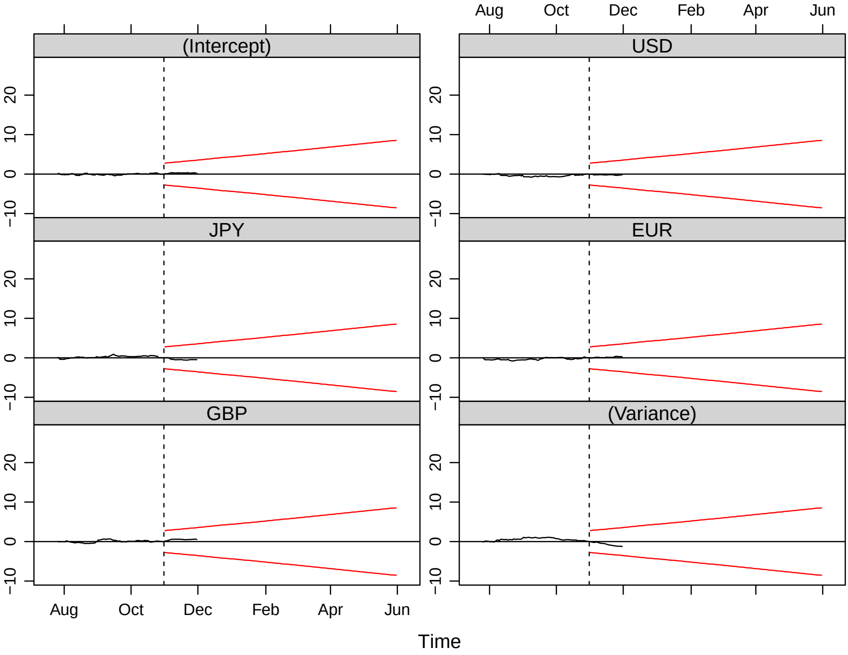
For CNY regression: Double maximum functional with boundary $b(t) = c \cdot t$ at $\alpha = 0.05$ for $T = 4$. Performed online on a web page in 2005/6.

Monitoring



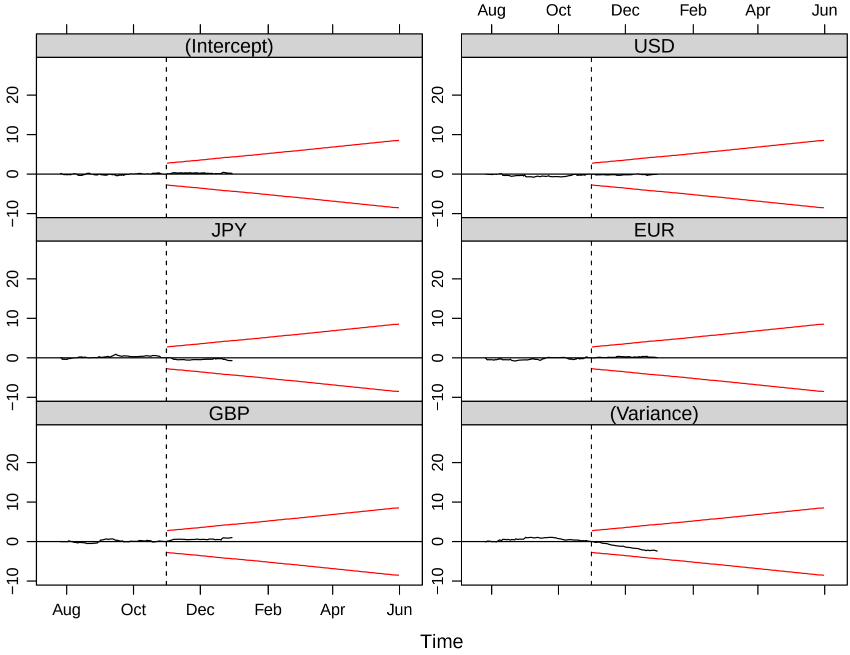
(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug
Oct
Dec
Feb
Apr
Jun
20
10
0
−10
Time

Monitoring



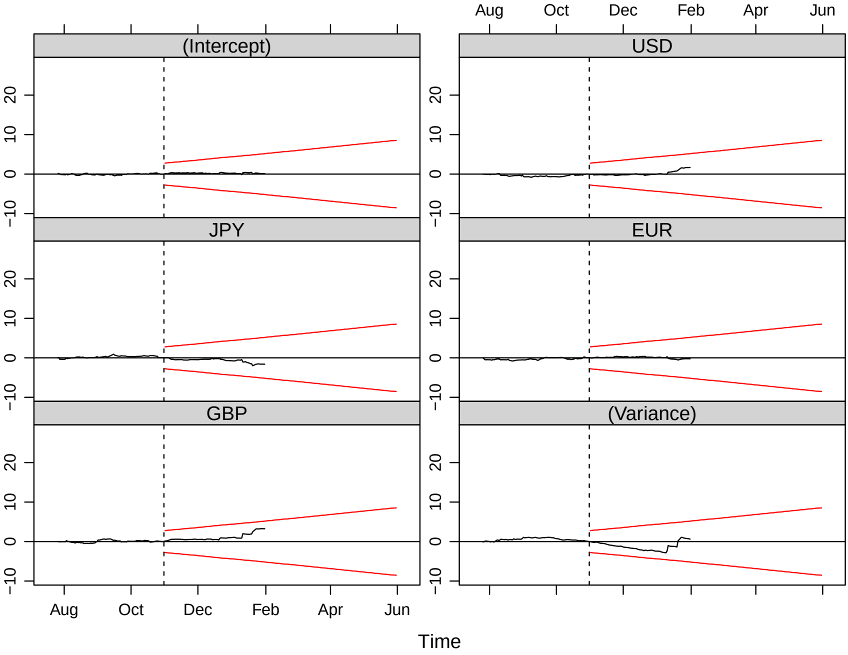
Aug
Oct
Dec
Feb
Apr
Jun
(Intercept)
USD
JPY
EUR
GBP
(Variance)
20
10
0
−10
Time

Monitoring



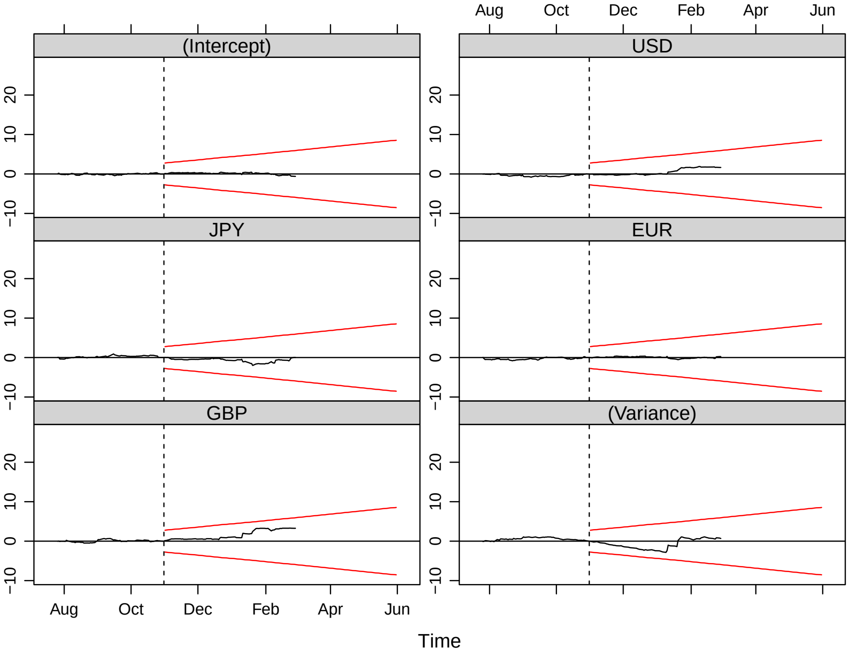
(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug
Oct
Dec
Feb
Apr
Jun
20
10
0
−10
Time

Monitoring



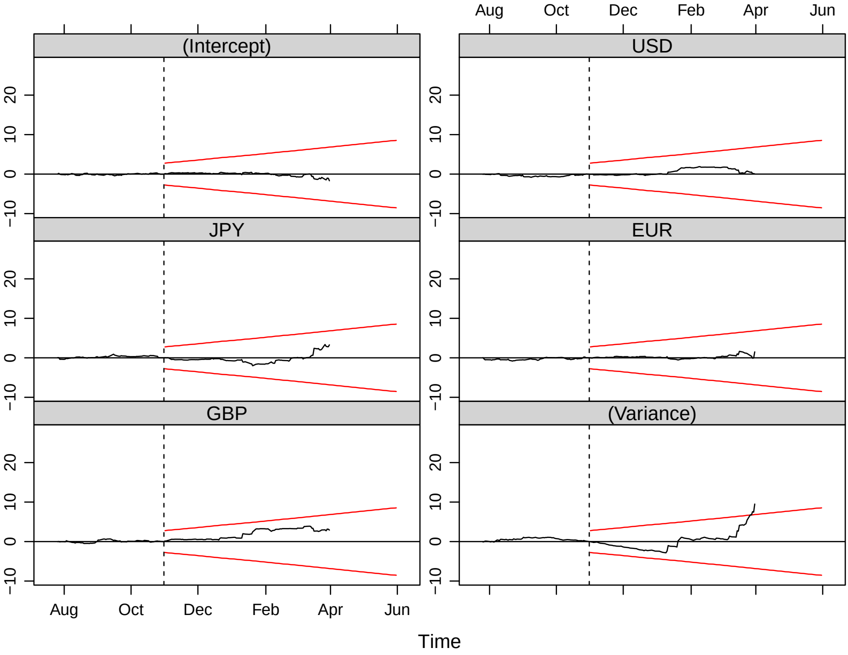
(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug
Oct
Dec
Feb
Apr
Jun
20
10
0
−10
Time

Monitoring



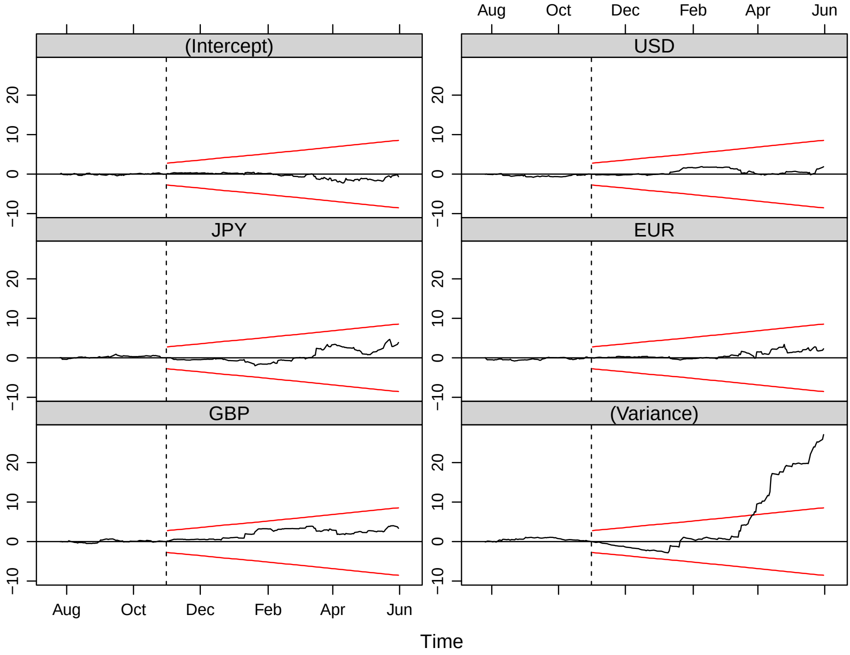
(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug
Oct
Dec
Feb
Apr
Jun
20
10
0
−10
Time

Monitoring



(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug
Oct
Dec
Feb
Apr
Jun
20
10
0
−10
Time

Monitoring



(Intercept)
USD
JPY
EUR
GBP
(Variance)
Aug
Oct
Dec
Feb
Apr
Jun
20
10
0
−10
Time

Monitoring

Results:

- This signals a clear increase in the error variance.
- The change is picked up by the monitoring procedure on 2006-03-27.
- The other regression coefficients did not change significantly, signalling that they are not part of the basket peg.
- Using data from an extended period up to 2009-07-31, we fit a segmented model to determine where and how the model parameters changed.

Dating

Segmented regression model: A stable model with parameter vector $\theta^{(j)}$ holds for the observations in segment j with $i = i_{j-1} + 1, \dots, i_j$.

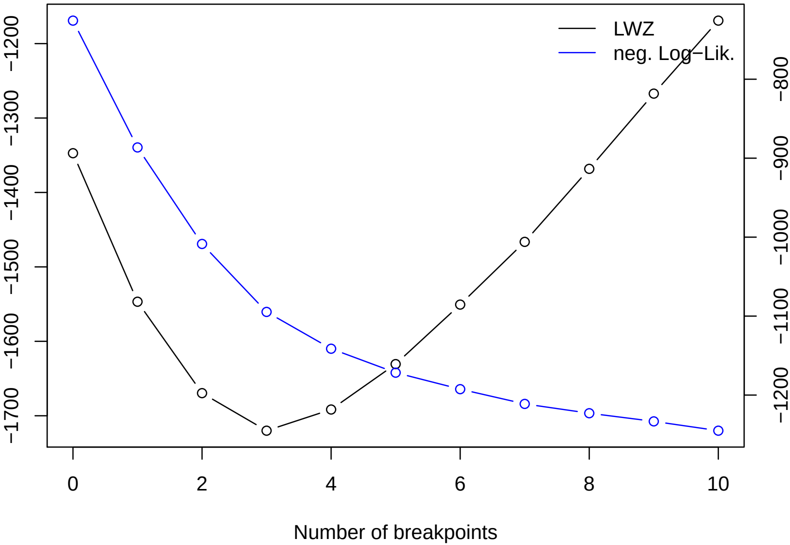
For CNY regression: Segmented (negative) log-likelihood from a normal model to capture changes in coefficients β and variance σ^2.

$$
\begin{aligned}
NLL(m) &= \sum_{j=1}^{m+1} \sum_{i=i_{j-1}+1}^{i_j} \Psi_{\mathrm{NLL}}\left(y_i, x_i, \hat{\beta}^{(j)}, \hat{\sigma}^{2,(j)}\right), \\
\Psi_{\mathrm{NLL}}(y_i, x_i, \beta, \sigma^2) &= -\log\left(\sigma^{-1}\phi\left(\frac{y_i - x_i^\top \beta}{\sigma}\right)\right).
\end{aligned}
$$

Model selection: Determine number of breaks via information criteria.

$$
\begin{aligned}
IC(m) &= 2 \cdot NLL(m) \; + \; \mathrm{pen} \cdot ((m+1)k + m), \\
\mathrm{pen}_{\mathrm{BIC}} &= \log(n), \\
\mathrm{pen}_{\mathrm{LWZ}} &= 0.299 \cdot \log(n)^{2.1}.
\end{aligned}
$$

Dating



LWZ
neg. Log−Lik.
−1200
−1300
−1400
−1500
−1600
−1700
−800
−900
−1000
−1100
−1200
0
2
4
6
8
10
Number of breakpoints

Dating

The estimated breakpoints and parameters are:

start/end	β_0	β_{USD}	β_{JPY}	β_{EUR}	β_{GBP}	σ	R^2
2005-07-26	$-$**0.005**	**0.999**	0.005	-0.015	0.007	0.028	0.998
2006-03-14	(0.002)	(0.005)	(0.005)	(0.017)	(0.008)		
2006-03-15	$-$**0.025**	**0.969**	-0.009	0.026	-0.013	0.106	0.965
2008-08-22	(0.004)	(0.012)	(0.010)	(0.023)	(0.012)		
2008-08-25	-0.015	**1.031**	-0.026	0.049	0.007	0.263	0.956
2008-12-31	(0.030)	(0.044)	(0.030)	(0.059)	(0.035)		
2009-01-02	0.001	**0.981**	0.008	-0.008	0.009	0.044	0.998
2009-07-31	(0.004)	(0.005)	(0.004)	(0.009)	(0.004)		

corresponding to

1. tight USD peg with slight appreciation,
2. slightly relaxed USD peg with some more appreciation,
3. slightly relaxed USD peg without appreciation,
4. tight USD peg without appreciation.

Application: Indian FX regimes

India: Expanding economy with a currency receiving increased interest over the last years.

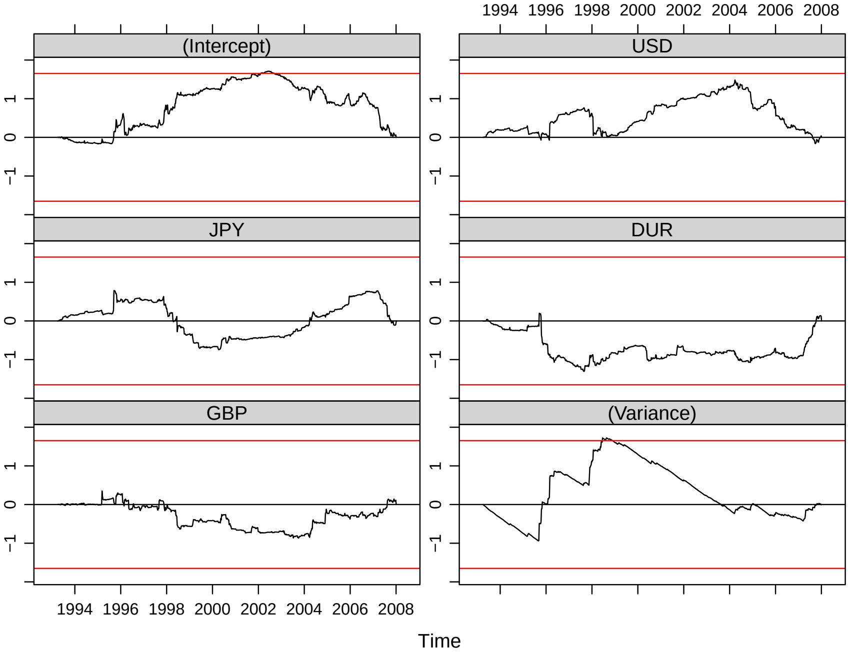
Here: Track evolution of INR FX regime since trading in INR began.

Data: Weekly returns from 1993-04-09 through to 2008-01-04 ($n = 770$).

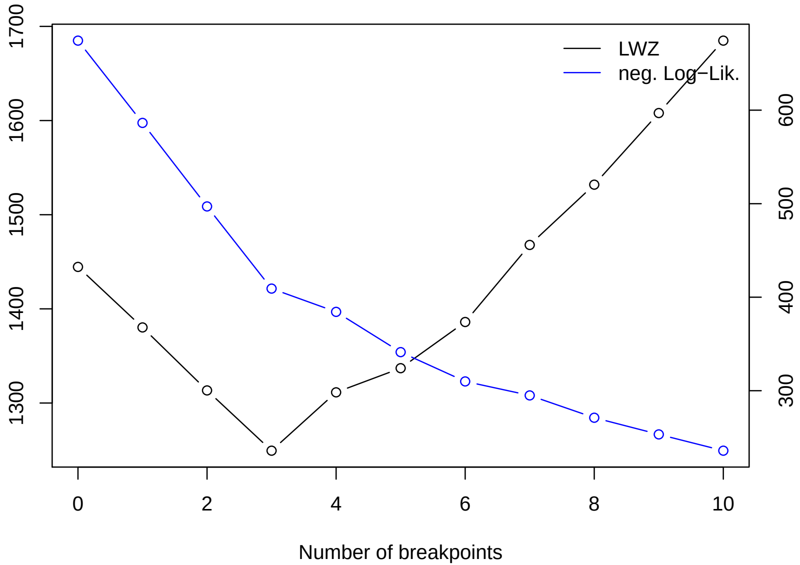
Testing: As multiple changes can be expected, assess stability of INR regime with the Nyblom-Hansen test, leading to 3.115 ($p < 0.005$). Alternatively, a MOSUM test could be used. The double maximum test has less power: 1.724 ($p = 0.031$).

Dating: Minimize segmented negative log-likelihood. Selection via LWZ yields 3 breakpoints.

Application: Indian FX regimes



Application: Indian FX regimes



Application: Indian FX regimes

The estimated breakpoints and parameters are:

start/end	β_0	β_{USD}	β_{JPY}	β_{DUR}	β_{GBP}	σ	R^2
1993-04-09	−0.006	**0.972**	0.023	0.011	0.020	0.157	0.989
1995-03-03	(0.017)	(0.018)	(0.014)	(0.032)	(0.024)		
1995-03-10	**0.161**	**0.943**	0.067	−0.026	0.042	0.924	0.729
1998-08-21	(0.071)	(0.074)	(0.048)	(0.155)	(0.080)		
1998-08-28	0.019	**0.993**	0.010	**0.098**	−0.003	0.275	0.969
2004-03-19	(0.016)	(0.016)	(0.010)	(0.034)	(0.021)		
2004-03-26	−0.058	**0.746**	**0.126**	**0.435**	**0.121**	0.579	0.800
2008-01-04	(0.042)	(0.045)	(0.042)	(0.116)	(0.056)		

corresponding to

1. tight USD peg,
2. flexible USD peg,
3. tight USD peg,
4. flexible basket peg.

Software

Implementation: All methods are freely available in the R system for statistical computing in the contributed packages *strucchange* and *fxregime* from the Comprehensive R Archive Network (`http://CRAN.R-project.org/`).

strucchange:

- Testing/monitoring/dating for OLS regressions.
- Object-oriented tools for testing of models with general M-type estimators.

fxregime:

- Testing/monitoring/dating of FX regressions based on normal (quasi-)ML.
- (Unexported) object-oriented tools for dating of models with additive objective function.

Summary

- Exchange rate regime analysis can be complemented by structural change tools.
- Both coefficients (currency weights) and error variance (fluctuation band) can be assessed using an (approximately) normal regression model.
- Estimation, testing, monitoring, and dating are all based on the same model, i.e., the same objective function.
- Model naturally leads to observation-wise measure of deviation. Alternative of interest drives choice of aggregation across observations.
- Traditional significance tests can be complemented by graphical methods conveying timing and component affected by a structural change.

References

Zeileis A, Shah A, Patnaik I (2010). "Testing, Monitoring, and Dating Structural Changes in Exchange Rate Regimes." *Computational Statistics & Data Analysis*, **54**(6), 1696–1706. `doi:10.1016/j.csda.2009.12.005`.

Zeileis A (2005). "A Unified Approach to Structural Change Tests Based on ML Scores, *F* Statistics, and OLS Residuals." *Econometric Reviews*, **24**(4), 445–466. `doi:10.1080/07474930500406053`

Zeileis A, Leisch F, Kleiber C, Hornik K (2005). "Monitoring Structural Change in Dynamic Econometric Models." *Journal of Applied Econometrics*, **20**(1), 99–121. `doi:10.1002/jae.776`

Zeileis A, Kleiber C, Krämer W, Hornik K, (2003). "Testing and Dating of Structural Changes in Practice." *Computational Statistics & Data Analysis*, **44**(1–2), 109–123. `doi:10.1016/S0167-9473(03)00030-6`

Zeileis A, Leisch F, Hornik K, Kleiber C (2002). "*strucchange*: An R Package for Testing for Structural Change in Linear Regression Models." *Journal of Statistical Software*, **7**(2), 1–38. URL `http://www.jstatsoft.org/v07/i02/`.