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Monitoring Structural Change in Dynamic Econometric Models

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Model frame

Consider the linear regression model in a monitoring situation

$$y_i = x_i^\top \beta_i + u_i \qquad (i = 1, \ldots, n, \ldots),$$

where at time i:

- y_i — dependent variable,
- x_i — vector of k regressors,
- β_i — vector of k unknown regression coefficients,
- u_i — error term.

Model frame

It is assumed that the regression relationship is stable ($\beta_i = \beta_0$) during the history period $i = 1, \ldots, n$.

Null hypothesis:

$$H_0: \quad \beta_i = \beta_0 \qquad (i > n),$$

Alternative:

$$H_1: \quad \beta_i \neq \beta_0 \qquad \text{for some } (i > n).$$

Generalized fluctuation tests

- empirical fluctuation processes reflect fluctuation in
  - residuals
  - coefficient estimates
- theoretical limiting process is known
- choose boundaries which are crossed by the limiting process only with a known probability α.
- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large $\Rightarrow$ reject the null hypothesis.

Generalized fluctuation tests

- Chu, Stinchcombe, White (1996)

  Extension of fluctuation tests to the monitoring situation: processes based on recursive estimates and recursive residuals.

- Leisch, Hornik, Kuan (2000)

  Generalized framework for estimates-based tests for monitoring.
  Contains the test of Chu et al., and considered in particular moving estimates.

Generalized fluctuation tests

Processes based on estimates:

$$\hat{\beta}^{(i)} \quad = \quad \left(X_{(i)}{}^{\top} X_{(i)}\right)^{-1} X_{(i)}{}^{\top} y^{(i)}$$

Recursive estimates (RE) process:

$$Y_n(t) = \frac{i}{\hat{\sigma}\sqrt{n}} \, Q_{(n)}^{\frac{1}{2}} \, \left(\hat{\beta}^{(i)} - \hat{\beta}^{(n)}\right),$$

where $i = \lfloor k + t(n-k) \rfloor$ and $t \geq 0$.

Generalized fluctuation tests

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where $i = \lfloor k + t(n-k) \rfloor$ and $t \geq 0$.

Moving estimates (ME) process:

$$Z_n\left(t|\,h\right) = \frac{\lfloor nh \rfloor}{\hat{\sigma}\sqrt{n}}\, Q_{(n)}^{\frac{1}{2}}\ \left(\hat{\beta}^{(\lfloor nt \rfloor - \lfloor nh \rfloor, \lfloor nh \rfloor)} - \hat{\beta}^{(n)}\right),$$

where $t \geq h$.

Generalized fluctuation tests

The empirical processes converge to a k-dimensional Brownian bridge or the increments thereof respectively.

The null hypothesis is rejected when the empirical processes cross the boundary

$$b_1(t) \quad = \quad \sqrt{t(t-1)\left[\lambda^2 + \log\left(\frac{t}{t-1}\right)\right]}$$

or

$$c(t) \quad = \quad \lambda \cdot \sqrt{\log_+ t}$$

respectively in the monitoring period $1 < t < T$ and λ determines the significance level of this procedure.

OLS-based processes

Processes based on OLS residuals:

$$\hat{u}_i \quad = \quad y_i - x_i^\top \hat{\beta}^{(n)}$$

OLS-based CUSUM process:

$$W_n^0(t) \quad = \quad \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \qquad (t \geq 0).$$

OLS-based processes

Processes based on OLS residuals:

$$\hat{u}_i \quad = \quad y_i - x_i^\top \hat{\beta}^{(n)}$$

OLS-based CUSUM process:

$$W_n^0(t) \quad = \quad \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \qquad (t \geq 0).$$

OLS-based MOSUM process:

$$M_n^0(t|h) \quad = \quad \frac{1}{\hat{\sigma}\sqrt{n}} \left(\sum_{i=\lfloor \eta t \rfloor - \lfloor nh \rfloor + 1}^{\lfloor \eta t \rfloor} \hat{u}_i \right) \qquad (t \geq h).$$

OLS-based processes

The limiting processes are the 1-dimensional Brownian bridge or the increments thereof respectively. Thus, the same boundaries can be used.

Advantage: ease of computation.

Rescaling

Kuan & Chen (1994):

Empirical size of (historical) estimates-based tests can be seriously distorted in dynamic models if the whole sample covariance matrix estimate

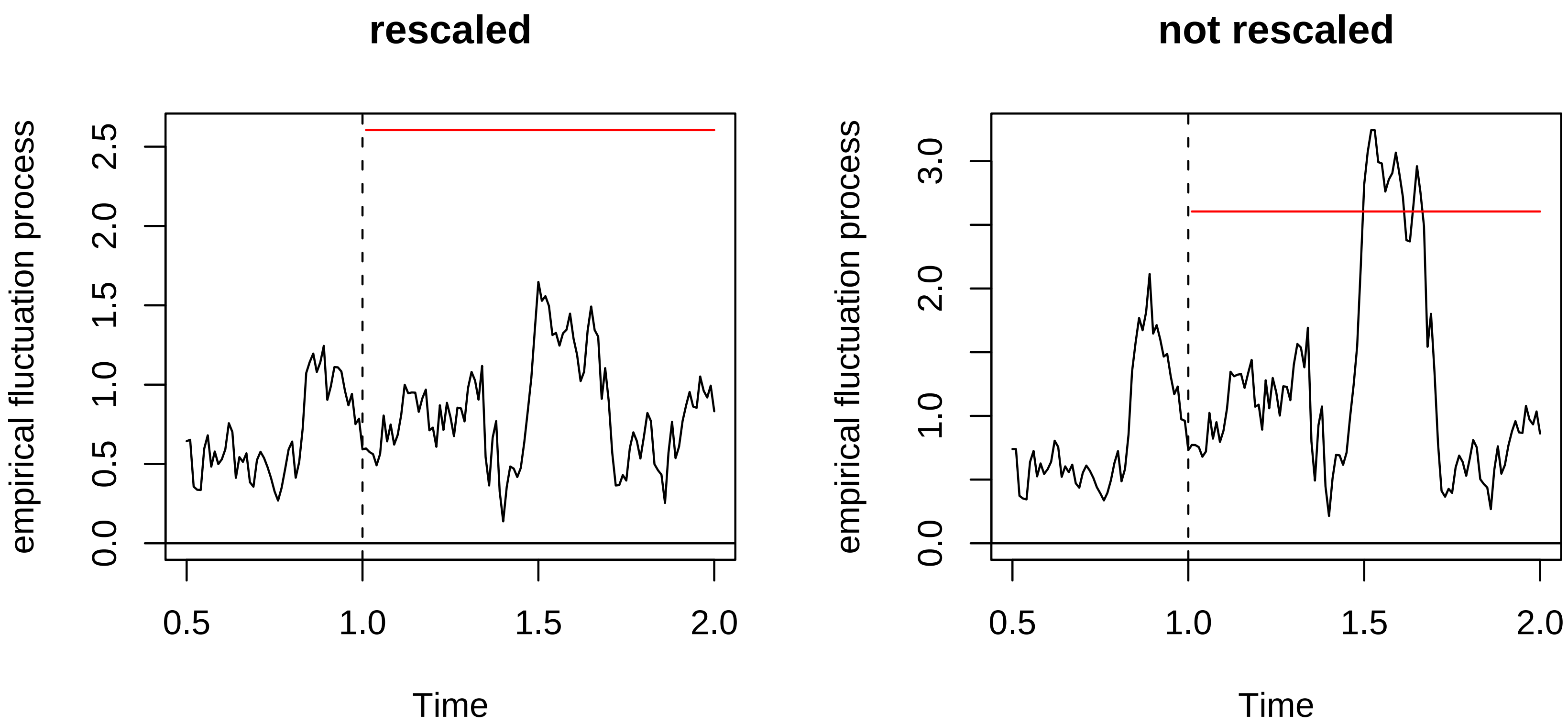
$$Q_{(n)} = 1/n \cdot X_{(n)}^\top X_{(n)}$$

is used to scale the fluctuation process.
Improvement: use $Q_{(i)}$ instead.

In a monitoring situation rescaling cannot improve the size of the RE test but it does so for the ME test!

Rescaling

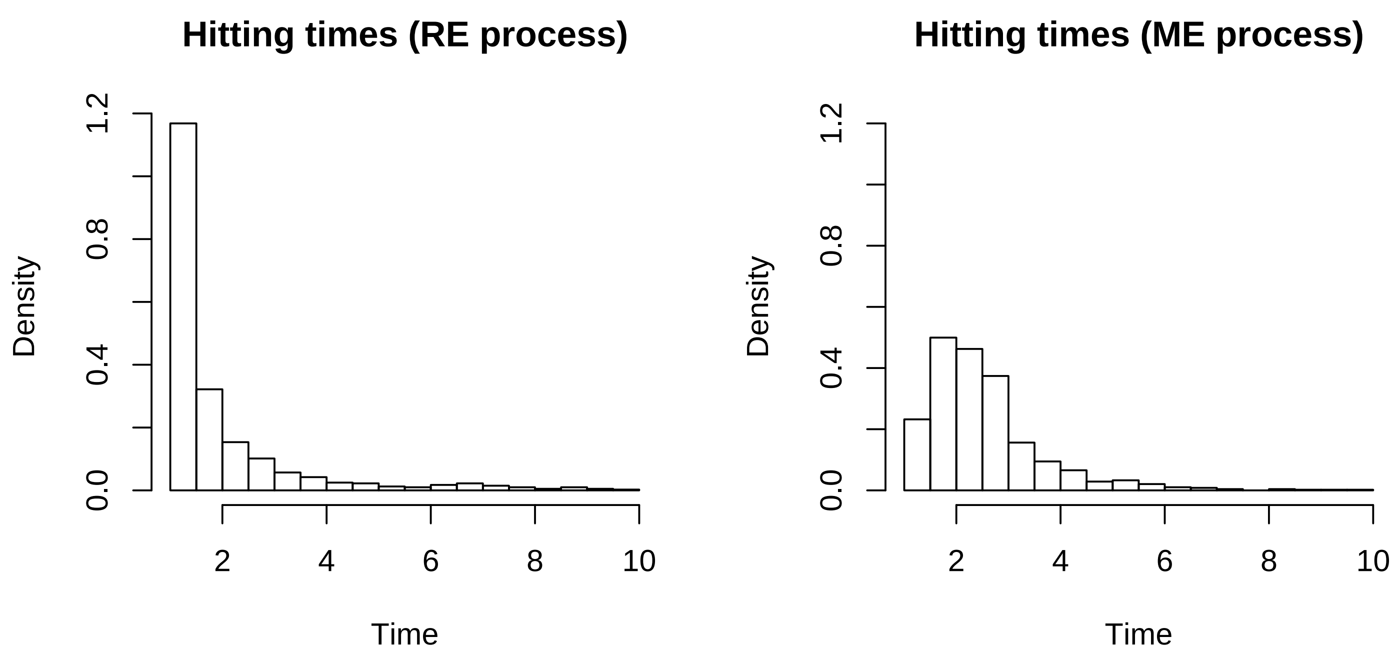
Example: AR(1) process with $\varrho = 0.9$ but *without* a shift:



Boundaries

The shape of the boundaries does not make a big difference under H_0, but determines the power for certain alternatives.

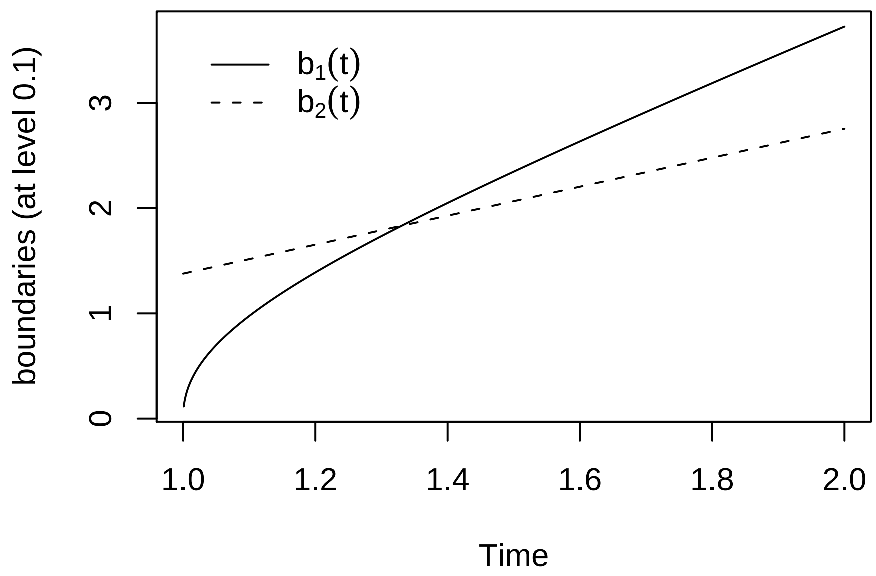
Standard boundaries:



Boundaries

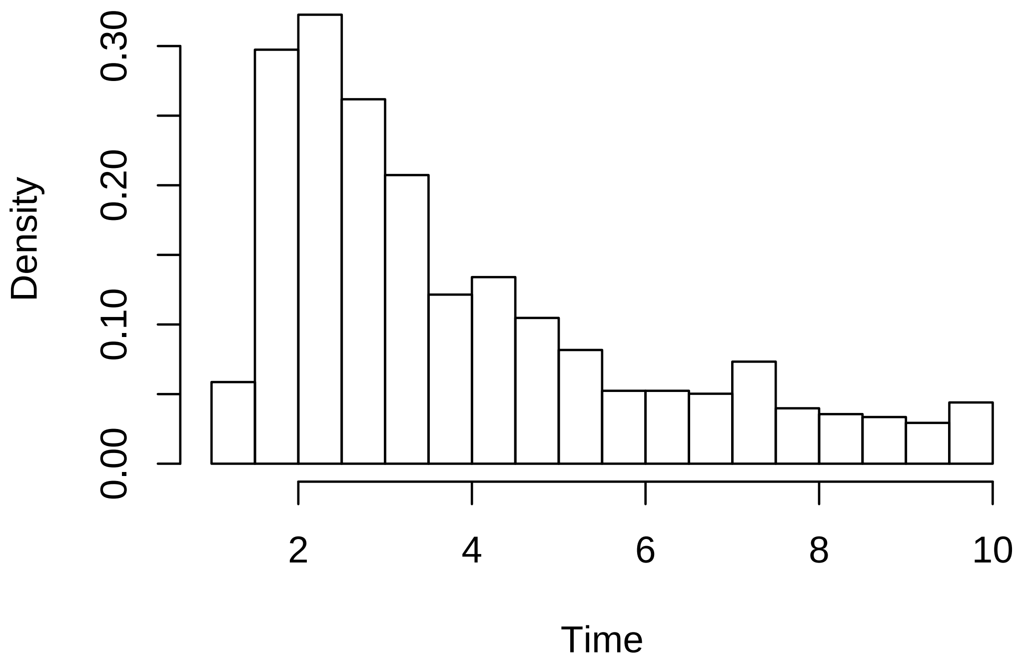
Consider a boundary with an offset in $t = 1$, but with the correct asymptotical growth rate $t \Rightarrow$ the simplest case:

$$b_2(t) \quad = \quad \lambda \cdot t.$$



Boundaries

This spreads the size much more evenly:



German M1 money demand

Lütkepohl, Teräsvirta, Wolters (1999) investigate the linearity and stability of German M1 money demand: stable regression relation for the time before the monetary union on 1990-06-01 but a clear structural instability afterwards.

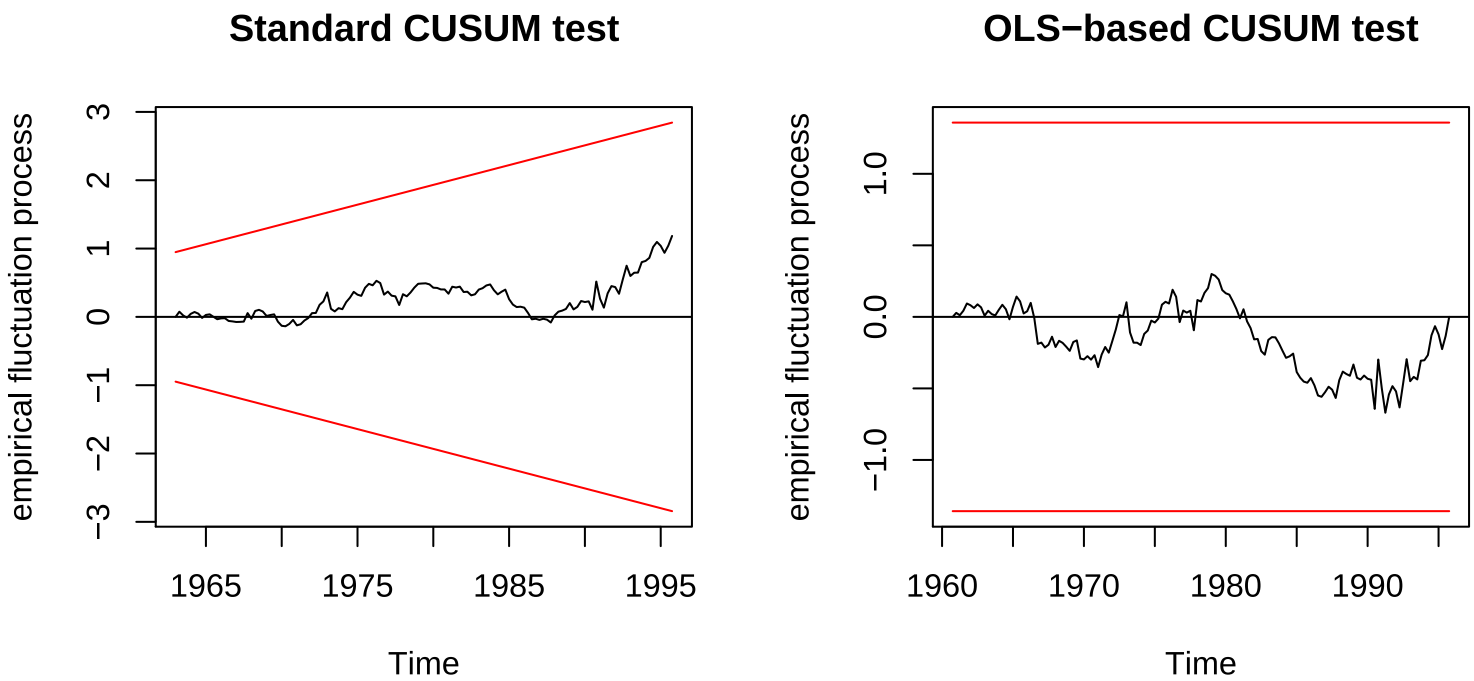
Data: seasonally unadjusted quarterly data, 1961(1) to 1995(4)

Error Correction Model (in logs) with variables:
M1 (real, per capita) m_t, price index p_t, GNP (real, per capita) y_t and long-run interest rate R_t:

$$\begin{aligned}\Delta m_t \;=\; & -0.30\Delta y_{t-2} - 0.67\Delta R_t - 1.00\Delta R_{t-1} - 0.53\Delta p_t \\ & -0.12 m_{t-1} + 0.13 y_{t-1} - 0.62 R_{t-1} \\ & -0.05 - 0.13Q1 - 0.016Q2 - 0.11Q3 + \hat{u}_t,\end{aligned}$$

German M1 money demand

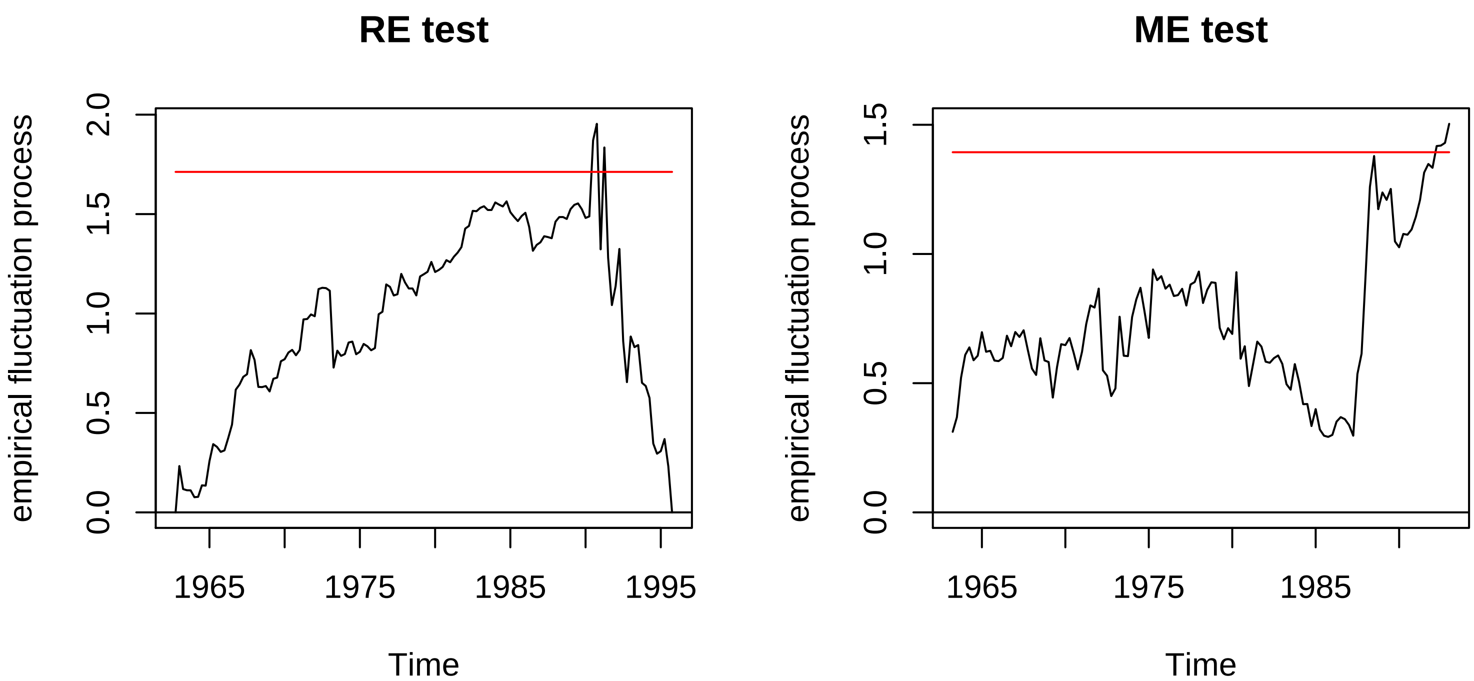
Historical OLS-based tests...do *not* discover shift:



The shift has an estimated angle of 90.27°.

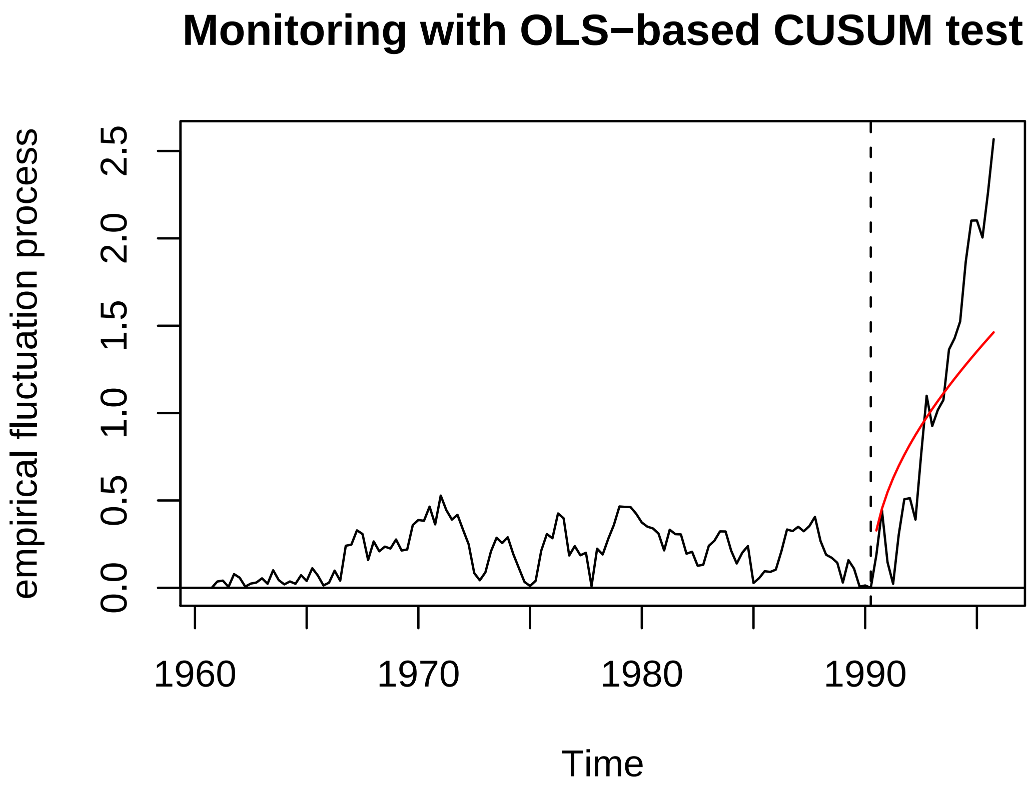
German M1 money demand

Historical estimates-based tests discover shift *ex post*:



German M1 money demand

Monitoring discovers shift *online*:



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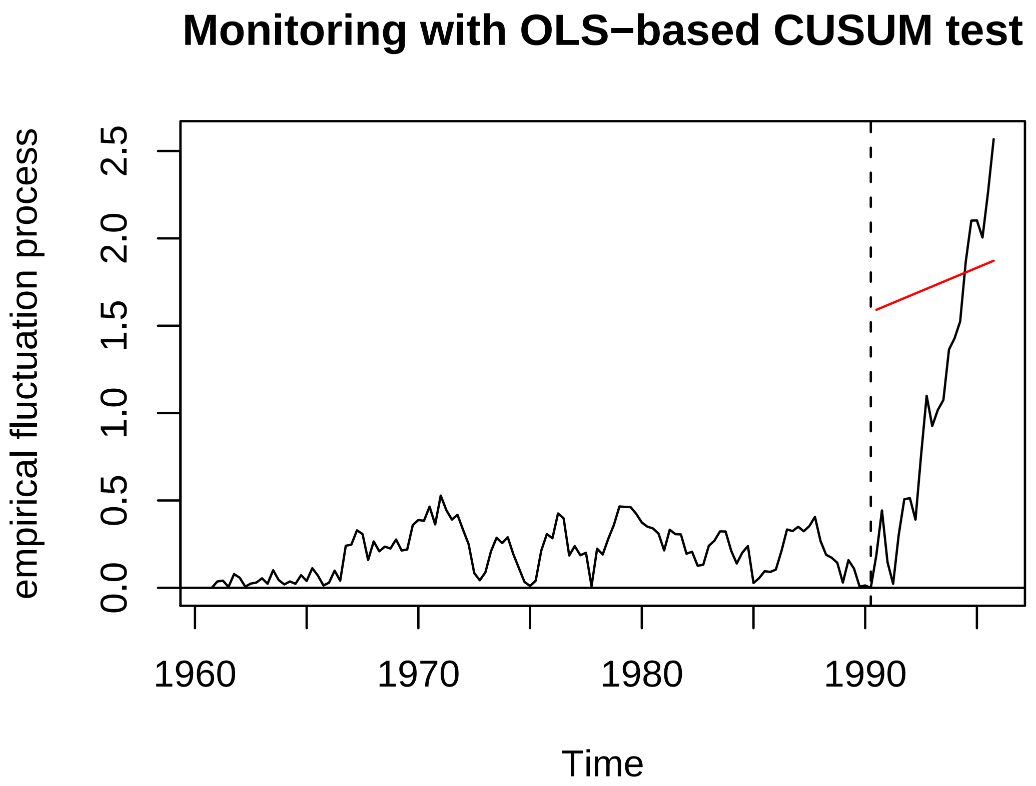

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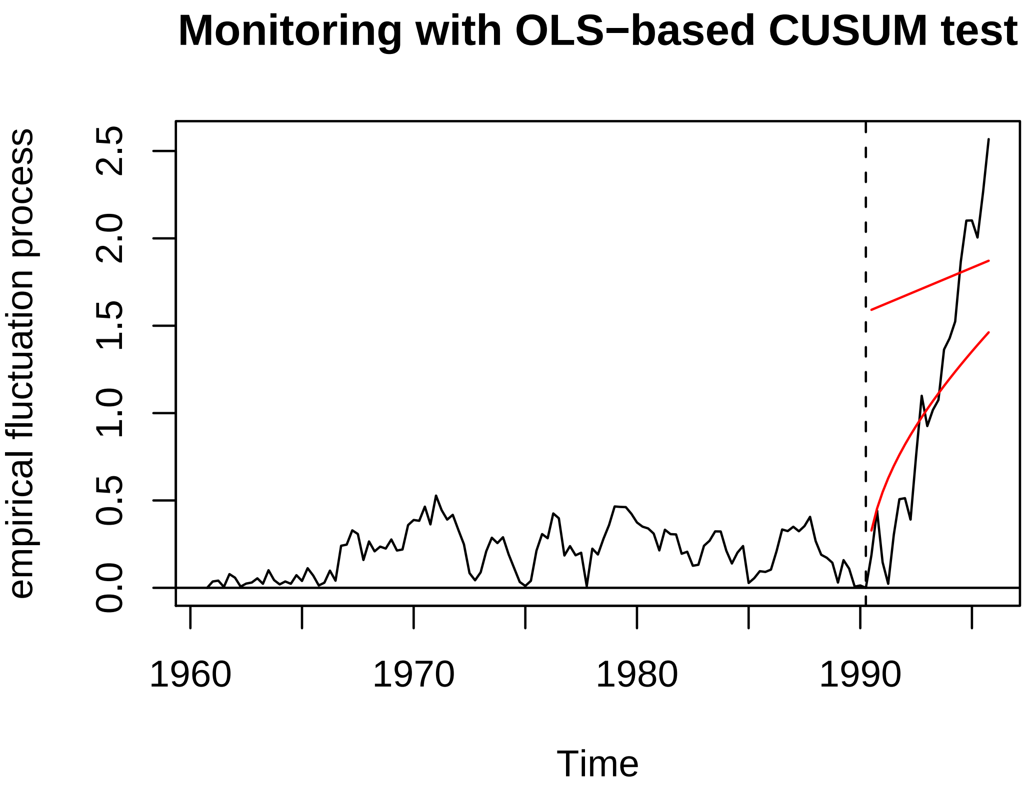


U.S. labor productivity

Hansen (2001) examines U.S. labor productivity in the manufacturing/durables sector

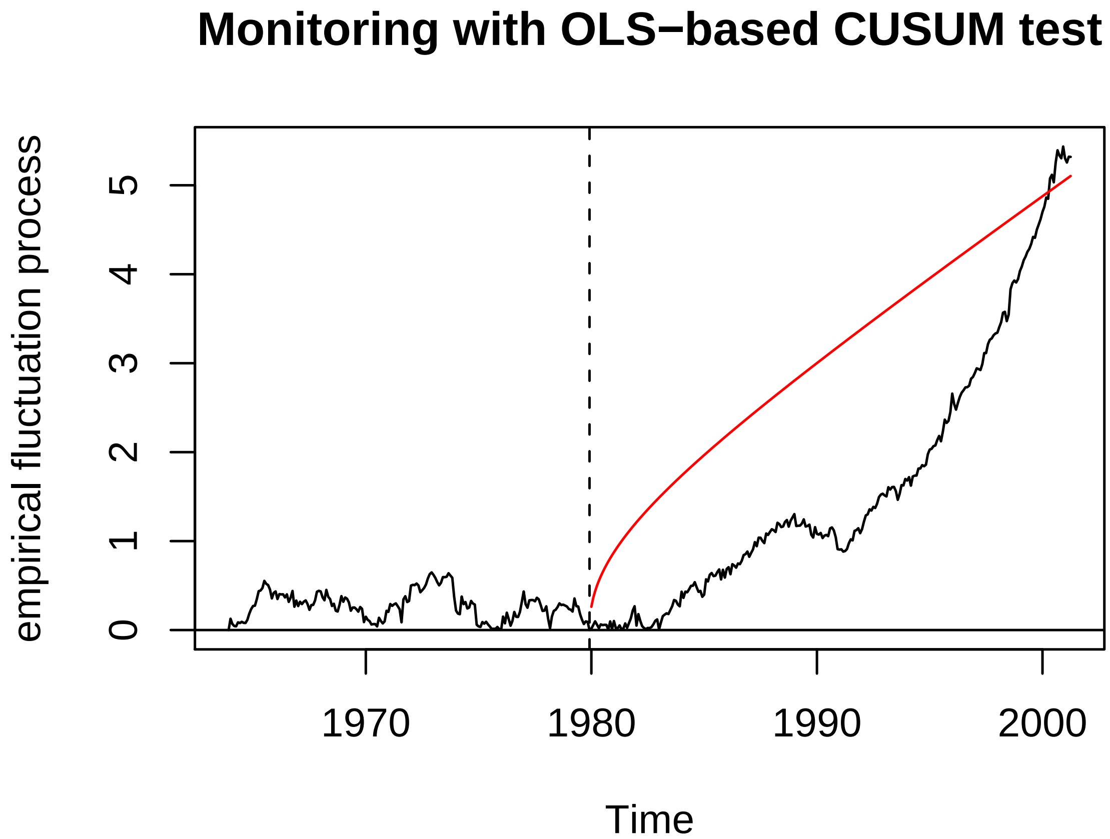
Monthly data, 1947(2) to 2001(4), AR(1) model.

Finds a clear structural change in about 1994 and two weaker changes in 1963 and 1982.

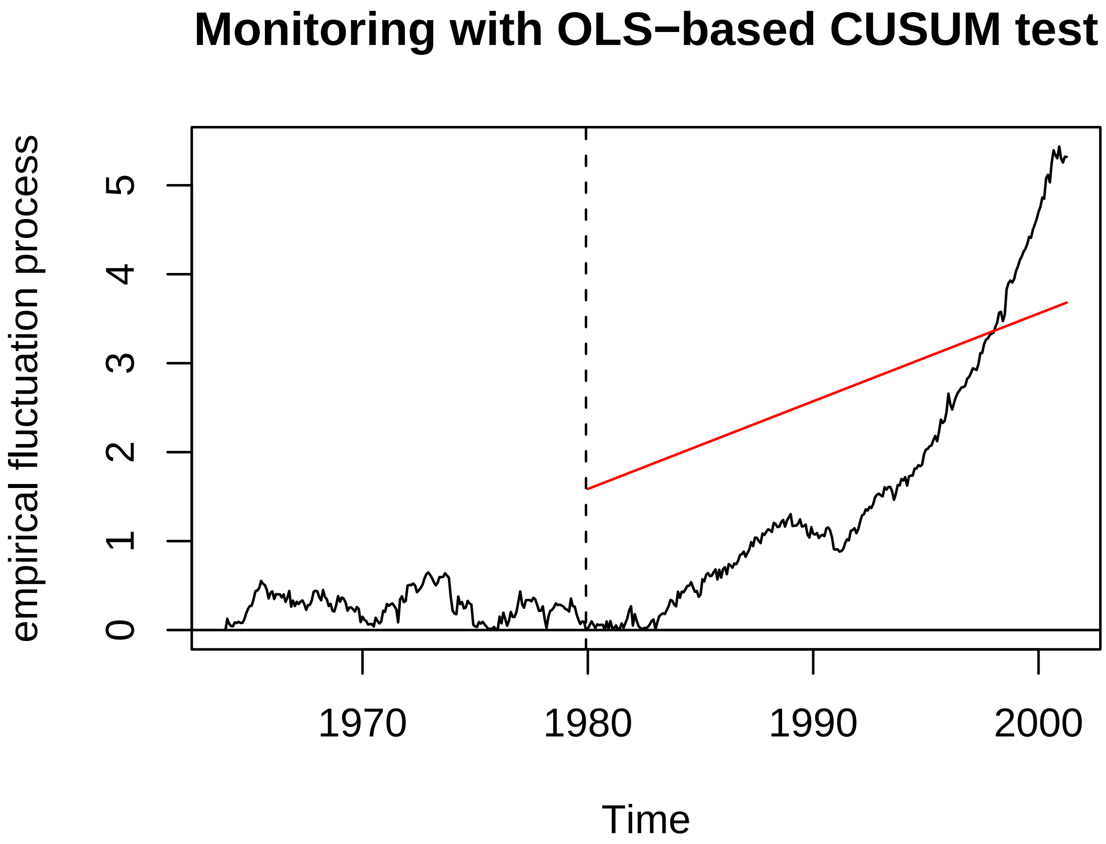
History period: 1964(1) to 1979(12)

$$x_t \quad = \quad 0.0025 - 0.186 x_{t-1} + \widehat{u}_t$$

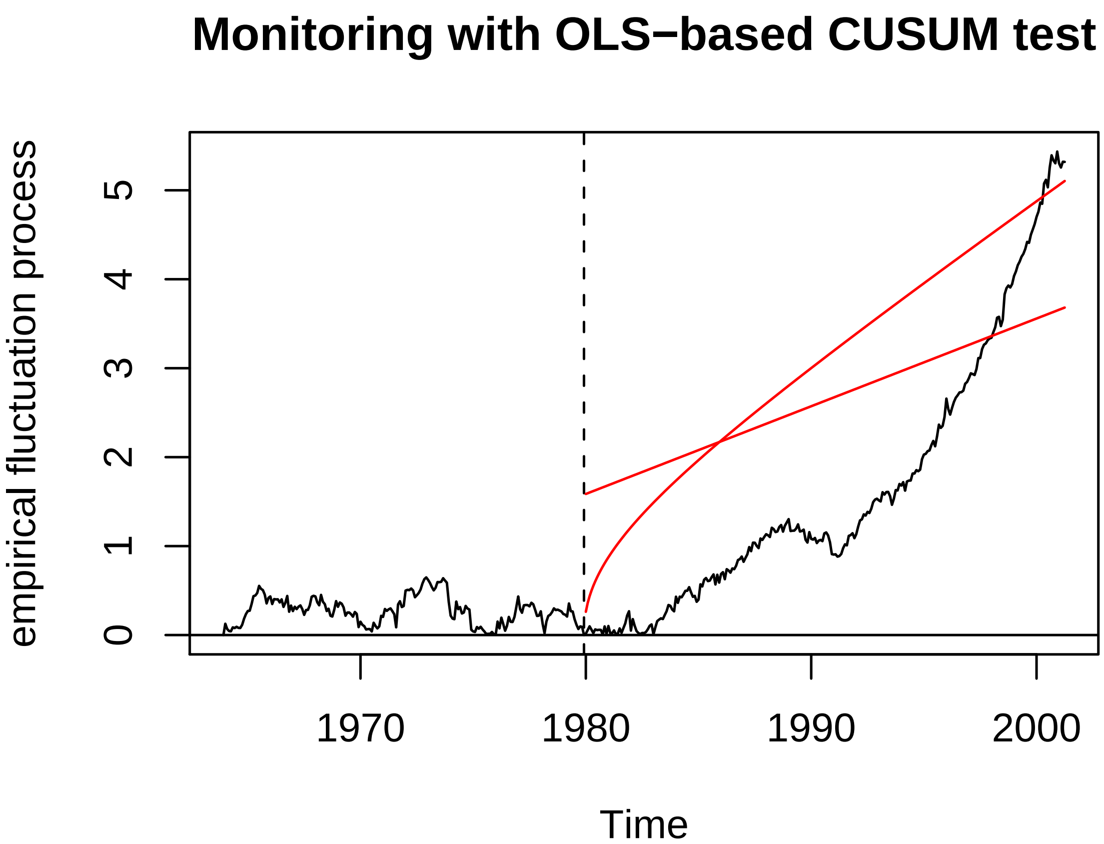
U.S. labor productivity



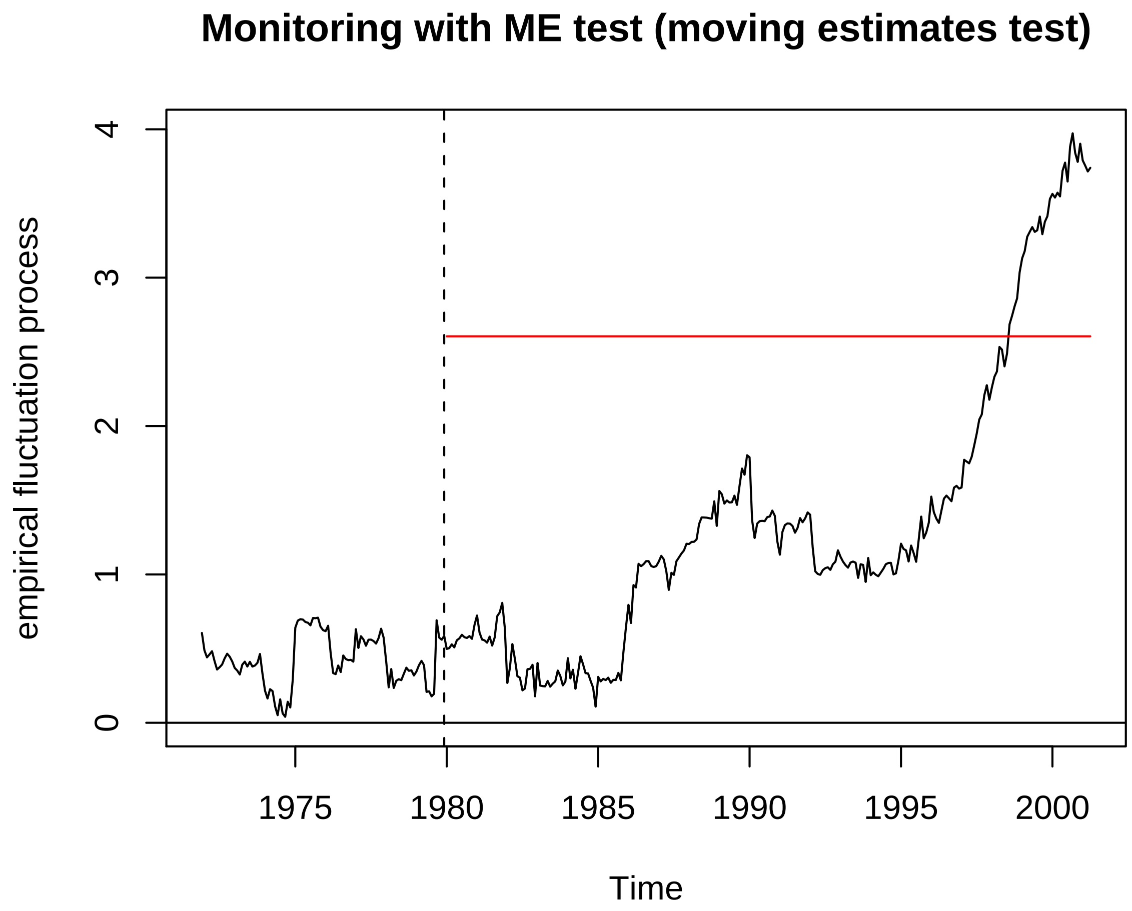
U.S. labor productivity



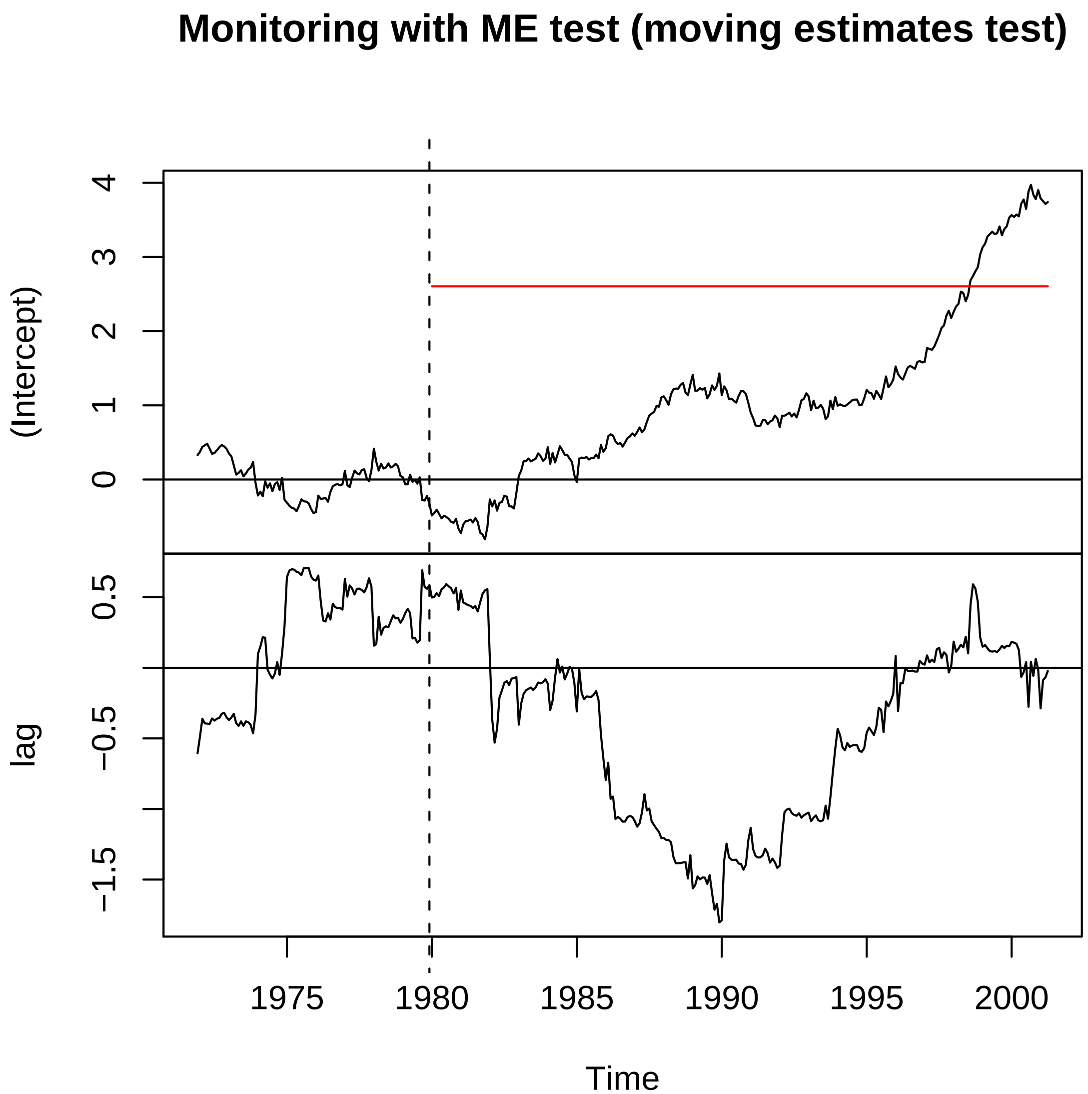
U.S. labor productivity



U.S. labor productivity



U.S. labor productivity



Software

All methods implemented in R

http://www.R-project.org/

in the contributed package `strucchange` availabe from the Comprehensive R Archive Network (CRAN):

http://cran.R-project.org/

documented in:

A. Zeileis, F. Leisch, K. Hornik, C. Kleiber (2002), "`strucchange`: An R Package for Testing for Structural Change in Linear Regression Models," *Journal of Statistical Software*, 7(2), 1–38.