



A Unified Approach to Testing, Monitoring, and Dating Structural Changes

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Overview

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- Summary

Model stability

Consider n ordered observations (y_i, x_i) for $i = 1, \dots, n$ in a regression setup which can be described by a regression model with parameter vector θ .

Ordering is typically with respect to time in time-series regressions, but could also be with respect to income, age, etc. in cross-section regressions.

Testing: Given that a model with parameter $\hat{\theta}$ has been estimated for these n observations, the question is whether this is appropriate or: ***Are the parameters stable or did they change through the sample period $i = 1, \dots, n$?***

Model stability

Monitoring: Given that a stable model could be established for these n observations, the question is whether it remains stable in the future or: ***Are incoming observations for $i > n$ still consistent with the established model or do the parameters change?***

Dating: Given that there is evidence for a structural change in $i = 1, \dots, n$, it might be possible that stable regression relationships can be found on subsets of the data. ***How many segments are in the data? Where are the breakpoints?***

Exchange rate regimes

The foreign exchange (FX) rate regime of a country determines how it manages its currency wrt foreign currencies. It can be

- *floating*: currency is allowed to fluctuate based on market forces,
- *pegged*: currency has limited flexibility when compared with a basket of currencies or a single currency,
- *fixed*: direct convertibility to another currency.

Problem: The *de facto* and *de jure* FX regime in operation in a country often differ.

⇒ Interest in methods for data-driven classification of FX regimes (see e.g., Reinhart and Rogoff, 2004).

The Chinese exchange rate regime

China gave up on a fixed exchange rate to the US dollar (USD) on 2005-07-21. The People's Bank of China announced that the Chinese yuan (CNY) would no longer be pegged to the USD but to a basket of currencies with greater flexibility.

This generated a lot of interest, both in the media and the scientific literature. Initially, little support could be found for these announcements (Frankel and Wei 2007).

Shah, Zeileis, Patnaik (2005) investigate the Chinese *de facto* FX regime based on a linear regression model for exchange rates (also called *Frankel-Wei regression*) using structural change methods.

Exchange rate regression

The popular workhorse for de facto FX regime classification is a linear regression model suggested by Haldane and Hall (1991) and Frankel and Wei (1994). It is based on log-returns of cross-currency exchange rates (with respect to some floating reference currency).

Fitting the model for CNY with regressors USD, JPY, EUR, and GBP (all wrt CHF) based on data up to 2005-10-31 ($n = 68$) shows that a plain USD peg is still in operation.

Exchange rate regression

Ordinary least squares (OLS) estimation gives:

$$\begin{aligned} \text{CNY}_i = & \underset{(0.004)}{0.005} + \underset{(0.009)}{0.9997} \text{USD}_i + \underset{(0.011)}{0.005} \text{JPY}_i \\ & - \underset{(0.027)}{0.014} \text{EUR}_i - \underset{(0.015)}{0.008} \text{GBP}_i + \hat{u}_i \end{aligned}$$

Only the USD coefficient is significantly different from 0 (but not from 1).

The error standard deviation is tiny with $\hat{\sigma} = 0.028$ leading to $R^2 = 0.998$.

Exchange rate regression

Questions:

1. Is this model for the period 2005-07-26 to 2005-10-31 stable or is there evidence that China kept changing its FX regime after 2005-07-26? (***testing***)
2. Depending on the answer to the first question:
 - Does the CNY stay pegged to the USD in the future (starting from November 2005)? (***monitoring***)
 - When and how did the Chinese FX regime change? (***dating***)

Exchange rate regression

In practice: Rolling regressions are often used to answer these questions by tracking the evolution of the FX regime in operation.

More formally: Structural change techniques can be adapted to the FX regression to estimate and test the stability of FX regimes.

Problem: Unlike many other linear regression models, the stability of the error variance (fluctuation band) is of interest as well.

Solution: Employ an (approximately) normal regression estimated by ML where the variance is a full model parameter.

Model frame

Consider a regression model with k -dimensional parameter θ for $y_i | x_i$. To fit the models to observations $i = 1, \dots, n$ an objective function $\Psi(y, x, \theta)$ is used such that

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{i=1}^n \Psi(y_i, x_i, \theta).$$

This can also be defined implicitly based on the corresponding score function (or estimating function) $\psi(y, x, \theta)$

$$\psi(y, x, \theta) = \frac{\partial \Psi(y, x, \theta)}{\partial \theta}$$

via

$$\sum_{i=1}^n \psi(y_i, x_i, \hat{\theta}) = 0.$$

Model frame

This class of M-estimators includes OLS and maximum likelihood (ML) estimation as well as IV, Quasi-ML, robust M-estimation and is closely related to GMM.

Under parameter stability and some mild regularity conditions, a central limit theorem holds

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, V(\theta_0)),$$

where the covariance matrix is

$$V(\theta_0) = \{A(\theta_0)\}^{-1} B(\theta_0) \{A(\theta_0)\}^{-1}$$

and A and B are the expectation of the derivative of ψ and its variance respectively.

Model frame

For the standard linear regression model

$$y_i = x_i^\top \beta + u_i$$

with coefficients β and error variance σ^2 one can either treat σ^2 as a nuisance parameter $\theta = \beta$ or include it as $\theta = (\beta, \sigma^2)$.

In the former case, the estimating functions are $\psi = \psi_\beta$

$$\psi_\beta(\mathbf{y}, \mathbf{x}, \beta) = (\mathbf{y} - \mathbf{x}^\top \beta) \mathbf{x}$$

and in the latter case, they have an additional component

$$\psi_{\sigma^2}(\mathbf{y}, \mathbf{x}, \beta, \sigma^2) = (\mathbf{y} - \mathbf{x}^\top \beta)^2 - \sigma^2.$$

and $\psi = (\psi_\beta, \psi_{\sigma^2})$. This is used for FX regressions.

Testing

To assess the stability of the fitted model with $\hat{\theta}$, we want to test the null hypothesis

$$H_0 : \theta_i = \theta_0 \quad (i = 1, \dots, n)$$

against the alternative that θ_i varies over “time” i .

Various patterns of deviation from H_0 are conceivable: single/multiple break(s), random walks, etc.

To test this null hypothesis, the basic idea is to assess whether the empirical estimating functions $\hat{\psi}_i = \psi(y_i, x_i, \hat{\theta})$ deviate systematically from their theoretical zero mean.

Testing

- empirical fluctuation processes reflect fluctuation in
 - estimating functions
 - F statistics
 - parameter estimates
 - recursive residuals
- theoretical limiting process is known
- choose boundaries which are crossed by the limiting process (or some functional of it) only with a known probability α .
- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large \Rightarrow reject the null hypothesis.

Testing

To capture systematic deviations the empirical fluctuation process of scaled cumulative sums of empirical estimating functions is computed:

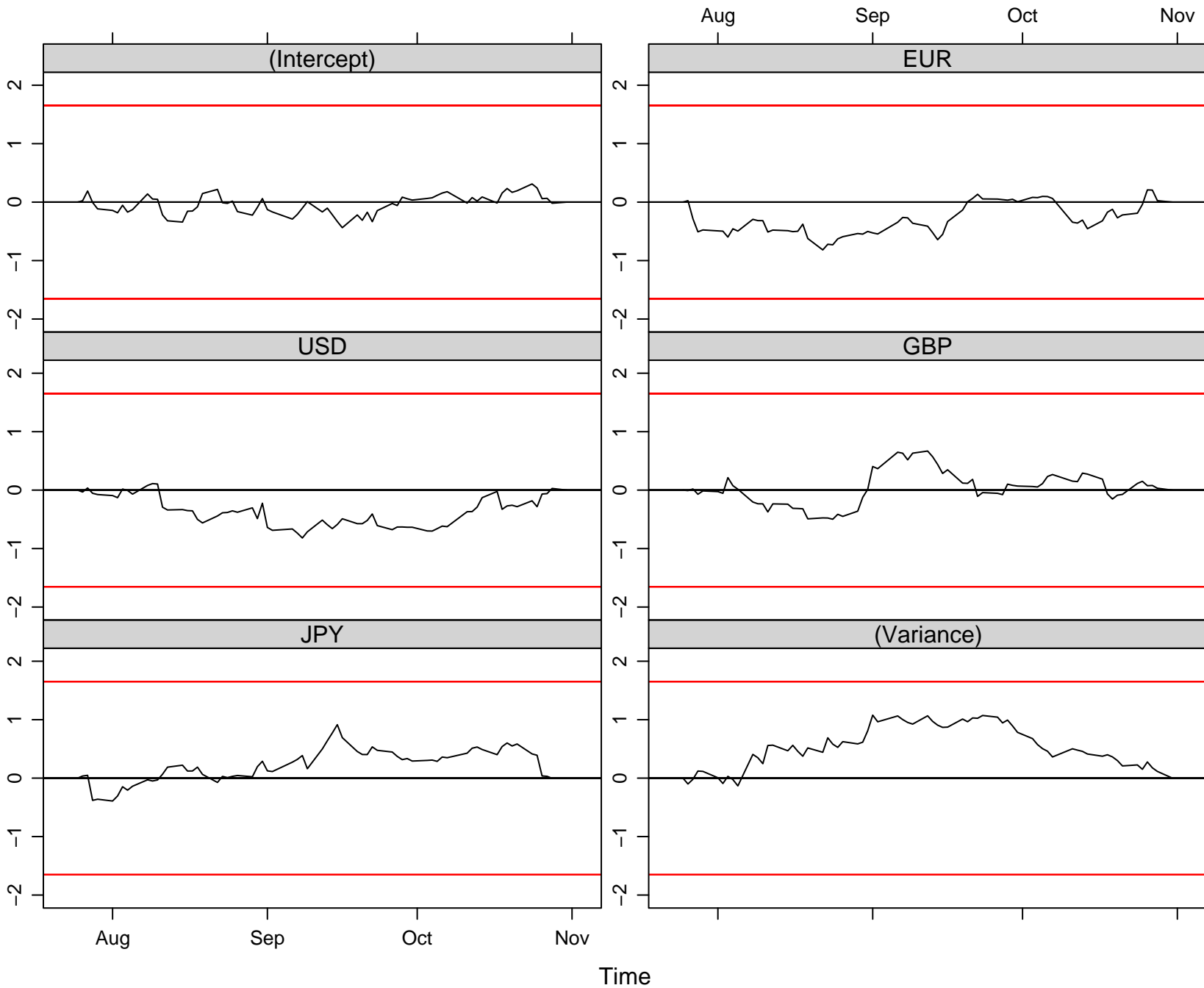
$$efp(t) = \hat{B}^{-1/2} n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \hat{\psi}_i \quad (0 \leq t \leq 1).$$

Under H_0 the following functional central limit theorem (FCLT) holds:

$$efp(\cdot) \xrightarrow{d} W^0(\cdot),$$

where W^0 denotes a standard k -dimensional Brownian bridge.

Testing



Testing

In empirical samples, $efp(\cdot)$ is a $k \times n$ array. For significance testing, aggregate it to a scalar test statistic by a functional $\lambda(\cdot)$

$$\lambda \left(efp_j \left(\frac{i}{n} \right) \right),$$

where $j = 1, \dots, k$ and $i = 1, \dots, n$.

Typically, $\lambda(\cdot)$ can be split up into

- $\lambda_{\text{comp}}(\cdot)$ aggregating over components j (e.g., absolute maximum, Euclidian norm),
- $\lambda_{\text{time}}(\cdot)$ aggregating over time i (e.g., max, mean, range).

The limiting distribution is given by $\lambda(W^0)$ and can easily be simulated (or some closed form results are also available).

Testing

The generalized fluctuation test framework ...

“... includes formal significance tests but its philosophy is basically that of data analysis as expounded by Tukey. Essentially, the techniques are designed to bring out departures from constancy in a graphic way instead of parametrizing particular types of departure in advance and then developing formal significance tests intended to have high power against these particular alternatives.” (Brown, Durbin, Evans, 1975)

Testing

Aggregating over time first, yields k independent statistics, each associated with one parameter \Rightarrow ***component of change can be identified.***

Aggregating over components first, yields a single process that can be inspected for instabilities over time \Rightarrow ***timing of change can be identified.***

The only functional that allows for both interpretations is the double maximum functional

$$\max_{j=1,\dots,k} \max_{i=1,\dots,n} |efp_j(i/n)|$$

which is 1.078 for the Chinese FX regression ($p = 0.73$).

Testing

This class of generalized M-fluctuation tests is derived in Zeileis & Hornik (2007). It contains various well-known tests from the statistics and econometrics literature as special cases (Zeileis 2005).

OLS-based CUSUM test (Ploberger & Krämer 1992, 1996)

If the regression model contains an intercept the first component of the estimating functions corresponds to the OLS residuals

$$\psi(y, x, \theta)_1 = (y - x^\top \beta)$$

The process based on OLS residuals can capture changes in the conditional mean.

Testing

Nyblom-Hansen test (Nyblom 1989, Hansen 1992)

The test was designed for a random-walk alternative and employs a Cramér-von Mises functional.

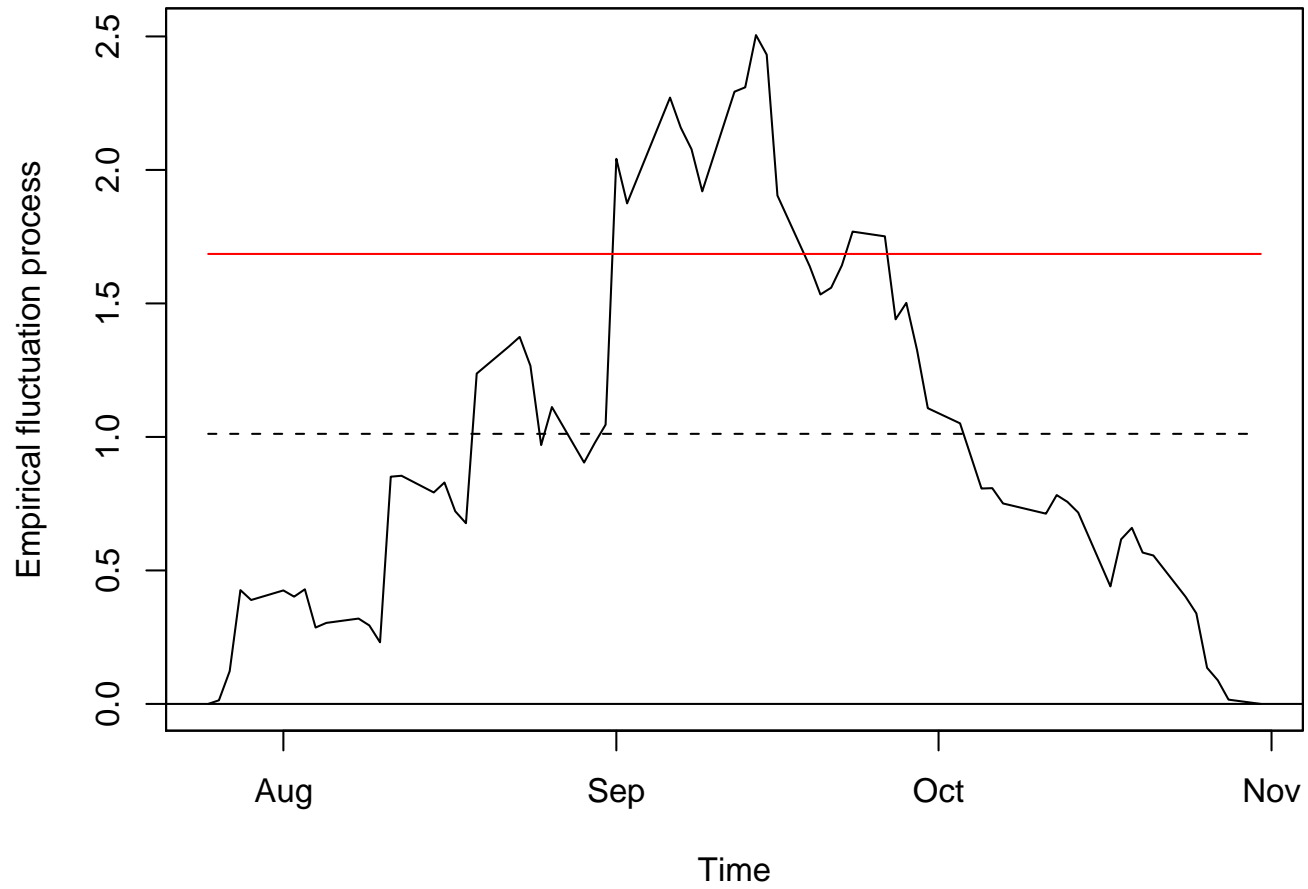
It aggregates $efp(\cdot)$ over the components first, using the squared Euclidian norm, and then over time, using the mean.

$$\frac{1}{n} \sum_{i=1}^n \left\| efp \left(\frac{i}{n} \right) \right\|_2^2.$$

For the Chinese FX regression this is 1.012 ($p = 0.364$).

Testing

Nyblom–Hansen test



Testing

supLM test (Andrews 1993)

The test was designed for a single shift alternative (with unknown timing) and employs the supremum of *LM* statistics for this alternative.

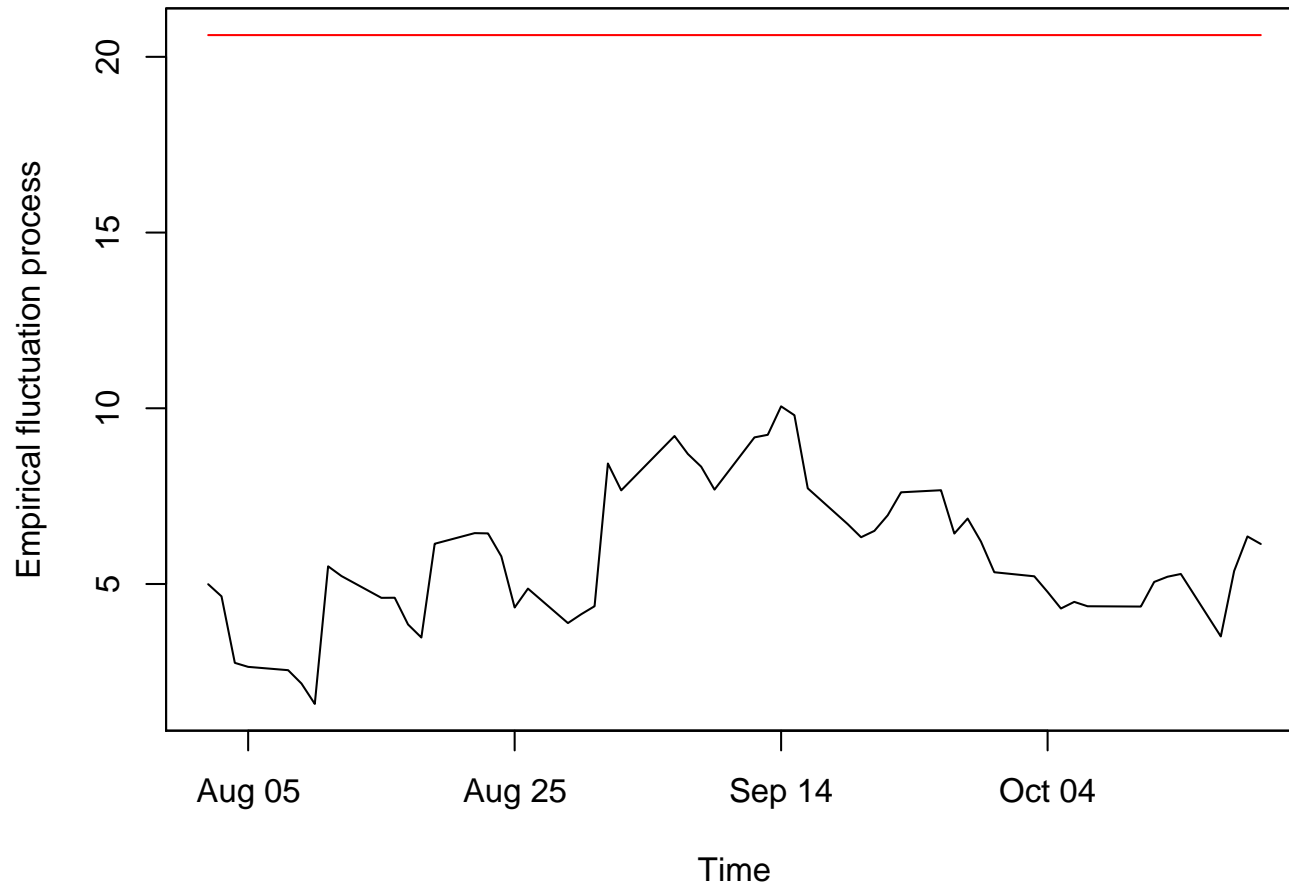
It aggregates *efp*(·) over the components first, using a weighted squared Euclidian norm, and then over time, using the maximum (over a compact interval $\Pi \subset [0, 1]$).

$$\sup_{t \in \Pi} LM(t) = \sup_{t \in \Pi} \frac{\|efp(t)\|_2^2}{t(1-t)}.$$

For the Chinese FX regression this is 10.055 ($p = 0.766$).

Testing

supLM test



Testing

Similarly, *aveLM* and *expLM* statistics can be computed which have some optimality properties (Andrews & Ploberger 1994).

The same type of processes can be derived based on Wald and LR statistics. These are not special cases of the M-fluctuation framework because they require re-estimation of the parameters on sub-samples. However, the asymptotic behaviour is the same.

Similarly, fluctuation processes can be based on recursive or rolling parameter estimates (Ploberger, Krämer, Kontrus 1989; Chu, Hornik, Kuan 1995) which are also not special cases but have the same asymptotic properties.

Testing

Furthermore, fluctuation processes can be computed from recursive residuals (Brown, Durbin, Evans 1975; Bauer & Hackl 1978; Krämer, Ploberger, Alt 1988) which are similar in spirit, but have slightly different asymptotic properties.

In practice, this multitude of processes and functionals is often a curse rather than a blessing. ***Which combination of process and functional should be used?***

Testing

Which process?

The M-fluctuation tests only depend on one fitted model and are hence very easy to compute and interpret \Rightarrow ideal for diagnostic checking, especially with no particular alternative in mind.

If a single shift alternative is plausible, it might be worth the effort to compute a sequence of Wald (or LR) statistics.

Recursive and rolling approaches are most plausible in a monitoring setup where a model should be simultaneously updated and tested.

Testing

Which functional?

For explorative purposes, double maximum tests are most attractive and allow identification of component and timing of changes.

For random walk or single shift alternatives respectively (that affect all parameters), the Nyblom-Hansen or $\text{sup}LM$ functionals are most suitable.

For multiple shift alternatives, statistics based on moving sums (increments of $efp(\cdot)$) perform very well.

Monitoring

Fluctuation tests can be applied sequentially to monitor regression models (Chu, Stinchcombe, White 1996; Leisch, Hornik, Kuan 2000; Zeileis *et al.* 2005).

Basic assumption: The model parameters are stable $\theta_i = \theta_0$ in the history period $i = 1, \dots, n$ ($0 \leq t \leq 1$).

To assess whether the model remains stable we want to test the null hypothesis

$$H_0 : \theta_i = \theta_0 \quad (i > n)$$

against the alternative that θ_i changes at some time in the future. The observations $i > n$ correspond to $t > 1$.

Monitoring

In the monitoring period $1 \leq t \leq T$:

- use the same empirical fluctuation processes,
- update $efp(t)$,
- re-compute $\lambda(efp(t))$.

For this sequential procedure not only a single critical value is needed, but a full boundary function $b(t)$ that satisfies

$$1 - \alpha = P(\lambda(W^0(t)) \leq b(t) \mid t \in [1, T])$$

Various boundary functions (or weighting functions) are conceivable (see Horváth *et al.* 2004; Zeileis *et al.* 2005) that can direct power to early or late changes or try to spread the power evenly.

Monitoring

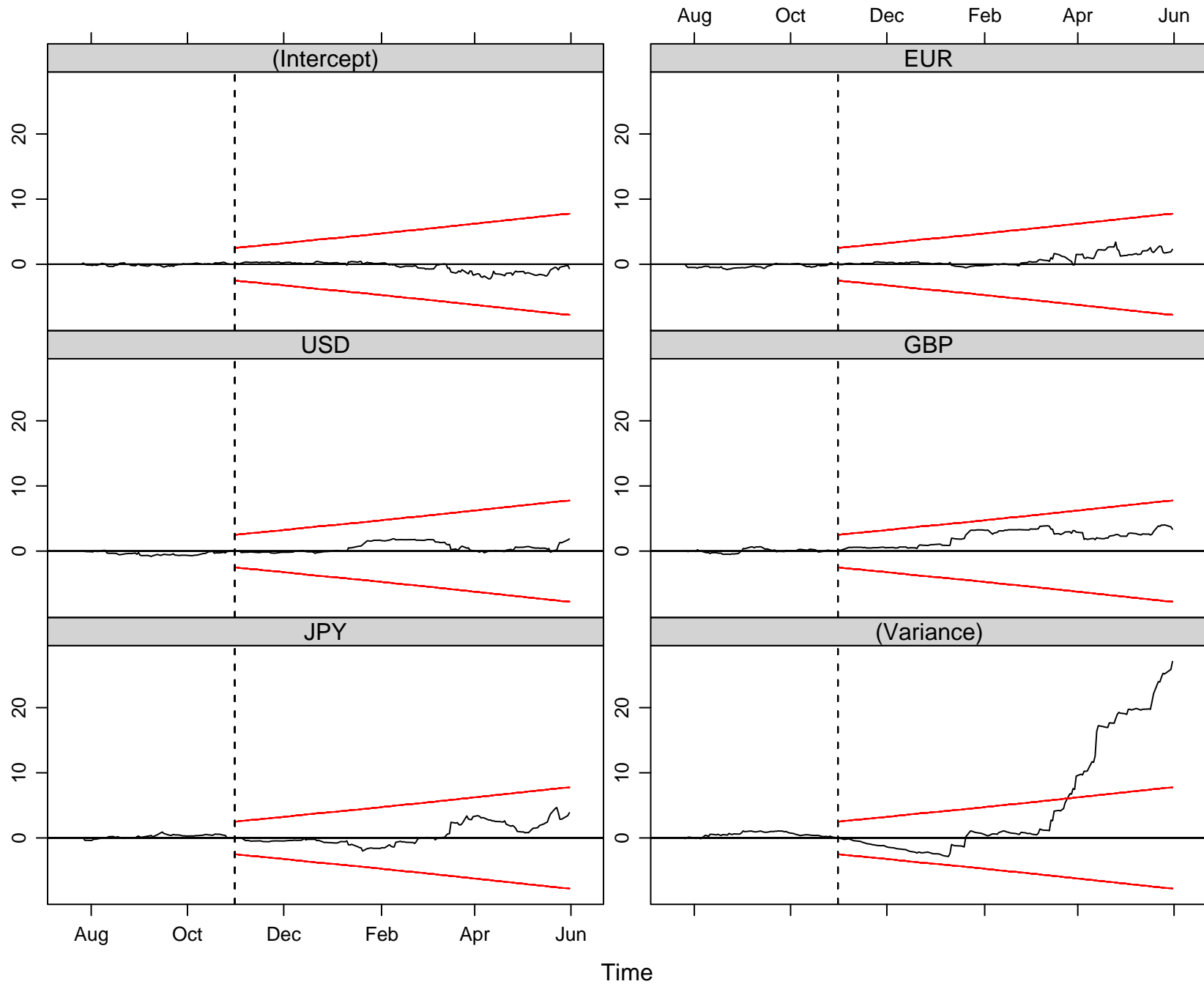
Here, we use again the double maximum functional and employ a simple boundary that spreads the power rather evenly:

$$b(t) = c \cdot t,$$

where c controls the significance level.

The fluctuation process used is the same M-fluctuation process used for the historical samples.

Monitoring



Monitoring

This signals a clear increase in the error variance.

The change is picked up by the monitoring procedure on 2006-03-27.

The other regression coefficients did not change significantly, signalling that they are not part of the basket peg.

Using data from the extended period up to 2007-12-28, we fit a segmented model to determine where and how the model parameters changed.

Dating

Segmented regression model: A stable model with parameter vector θ_j holds for the observations in $i = i_{j-1} + 1, \dots, i_j$. The segment index is $j = 1, \dots, m + 1$.

The set of m breakpoints $\mathcal{I}_{m,n} = \{i_1, \dots, i_m\}$ is called m -partition. Convention: $i_0 = 0$ und $i_{m+1} = n$.

The value of the segmented objective function Ψ is

$$PSI(i_1, \dots, i_m) = \sum_{j=1}^{m+1} psi(i_{j-1} + 1, i_j),$$
$$psi(i_{j-1} + 1, i_j) = \sum_{i=i_{j-1}+1}^{i_j} \Psi(y_i, x_i, \hat{\theta}^{(j)}).$$

Dating

where $psi(i_{j-1} + 1, i_j)$ is the minimal value of the objective function for the model fitted on the j th segment.

Dating tries to find

$$(\hat{i}_1, \dots, \hat{i}_m) = \underset{(i_1, \dots, i_m)}{\operatorname{argmin}} PSI(i_1, \dots, i_m)$$

over all partitions (i_1, \dots, i_m) with $i_j - i_{j-1} + 1 \geq \lfloor nh \rfloor \geq k$.

Bellman principle of optimality:

$$PSI(\mathcal{I}_{m,n}) = \min_{mn_h \leq i \leq n - n_h} [PSI(\mathcal{I}_{m-1,i}) + psi(i + 1, n)]$$

Dating

It is long-known that this problem can be solved by a dynamic programming algorithm of order $O(n^2)$ that essentially relies on a triangular matrix of $\psi(i, j)$ for all $1 \leq i < j \leq n$.

The algorithm has been re-discovered several times in the literature. A good overview is given by Bai & Perron (2003) for OLS-based dating, i.e., using the residual sum of squares as objective function

$$\Psi_{\text{RSS}}(\beta) = \sum_{i=1}^n (y_i - x_i^\top \beta).$$

Dating

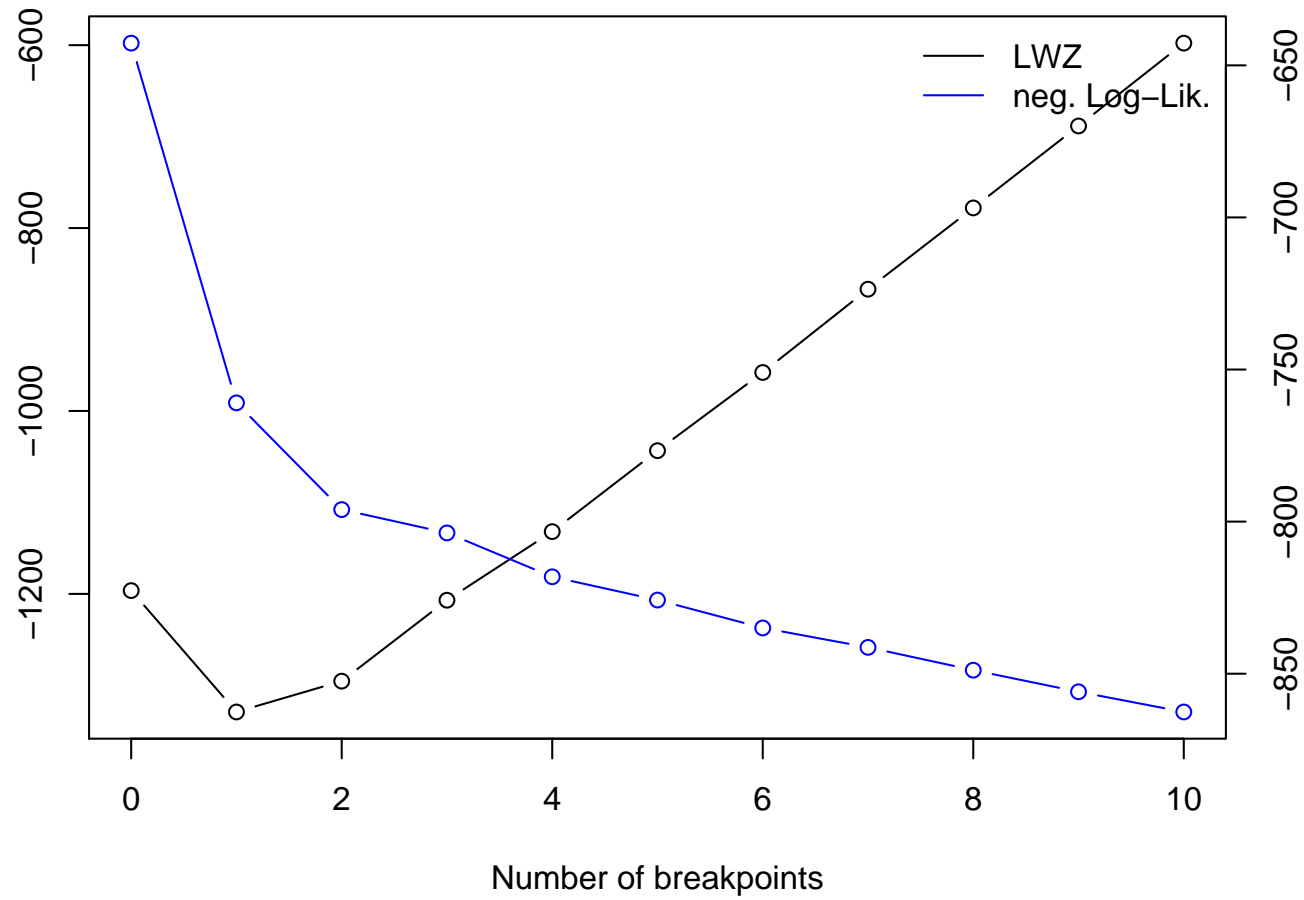
For the FX regression, the (negative) log-likelihood from a normal model also captures changes in the variance:

$$\Psi_{\text{NLL}}(\beta, \sigma) = - \sum_{i=1}^n \log \left(\sigma^{-1} \phi \left(\frac{y_i - x_i^\top \beta}{\sigma} \right) \right).$$

For a given number of breaks m , the optimal breaks be found. To determine the number of breaks, some model selection has to be done, e.g., via information criteria or sequential tests. Here, we use the LWZ criterion (modified BIC):

$$\begin{aligned} IC(m) &= 2 \cdot NLL(\mathcal{I}_{m,n}) + \text{pen} \cdot ((m + 1)k + m), \\ \text{pen}_{\text{BIC}} &= \log(n), \\ \text{pen}_{\text{LWZ}} &= 0.299 \cdot \log(n)^{2.1}. \end{aligned}$$

Dating



Dating

The estimated breakpoint (maximizing the segmented likelihood) is 2006-03-14.

The corresponding parameter estimates are

start/end	β_0	β_{USD}	β_{JPY}	β_{EUR}	β_{GBP}	σ	R^2
2005-07-26 2006-03-14	-0.005 (0.002)	0.999 (0.005)	0.005 (0.005)	-0.015 (0.017)	0.007 (0.008)	0.028	0.998
2006-03-15 2007-12-28	-0.020 (0.005)	0.976 (0.013)	-0.017 (0.011)	0.034 (0.029)	-0.010 (0.016)	0.096	0.960

and correspond to a

- very tight USD peg,
- slightly relaxed USD peg (with some more appreciation).

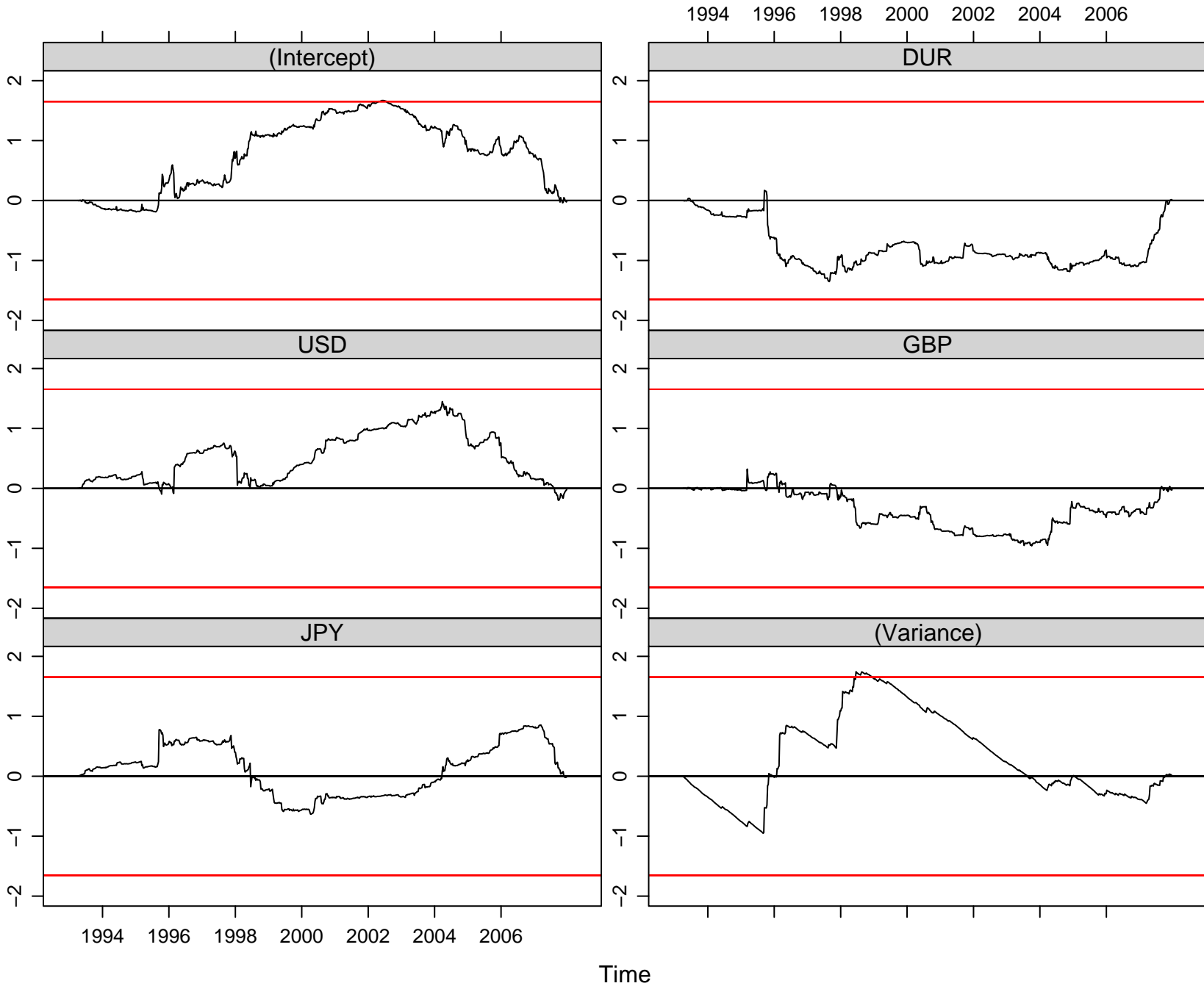
Application: Indian FX regimes

India also has an expanding economy with a currency receiving increased interest over the last years. We track the evolution of the INR FX regime since trading in the INR began.

Using weekly returns from 1993-04-09 through to 2007-12-28 (yielding $n = 769$ observations), we fit a single FX regression using the same basket as above.

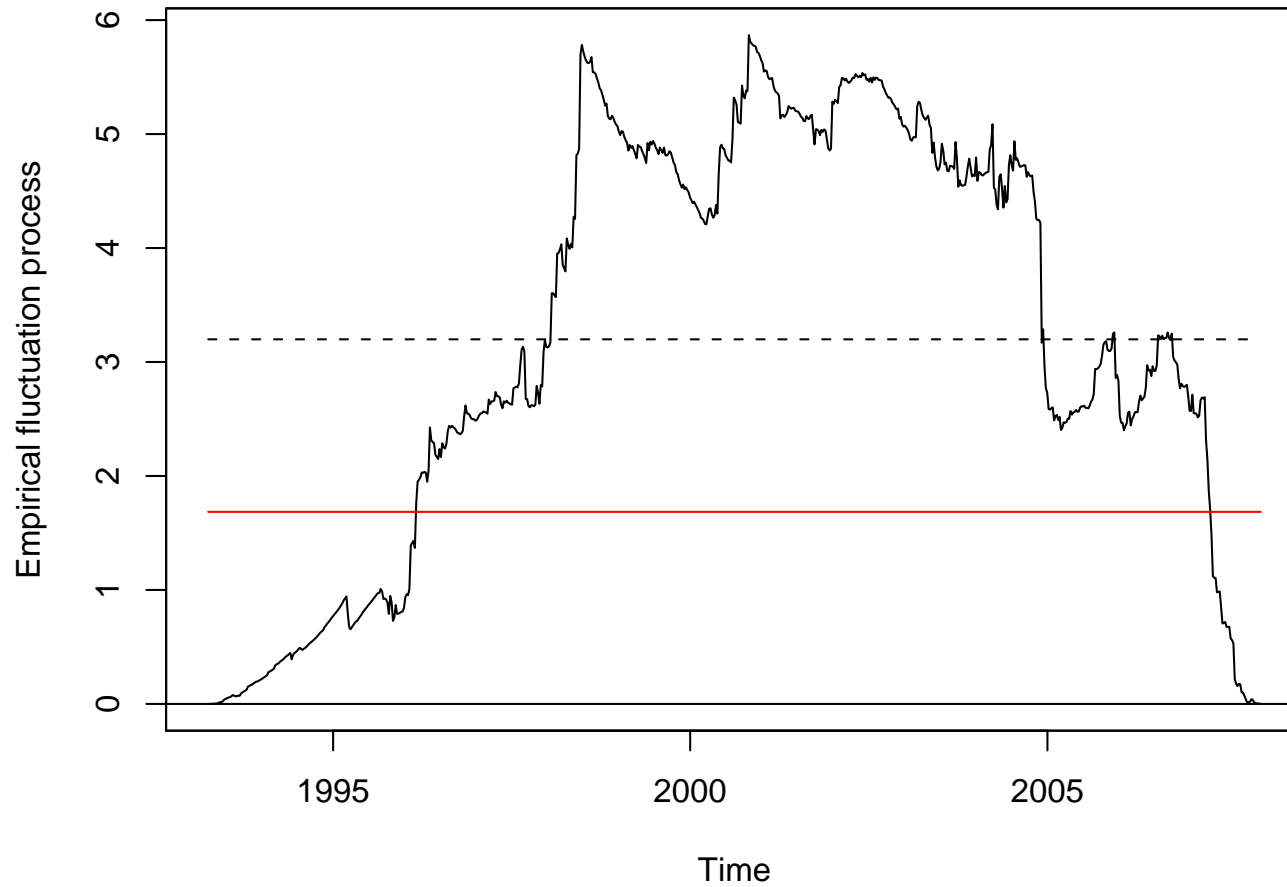
As we would expect multiple changes, we assess its stability with the Nyblom-Hansen test, leading to a test statistic of 3.199 ($p < 0.005$). Alternatively, a MOSUM test could be used. The double maximum test has less power, resulting in a test statistic of 1.744 ($p = 0.027$).

Application: Indian FX regimes



Application: Indian FX regimes

Nyblom–Hansen test



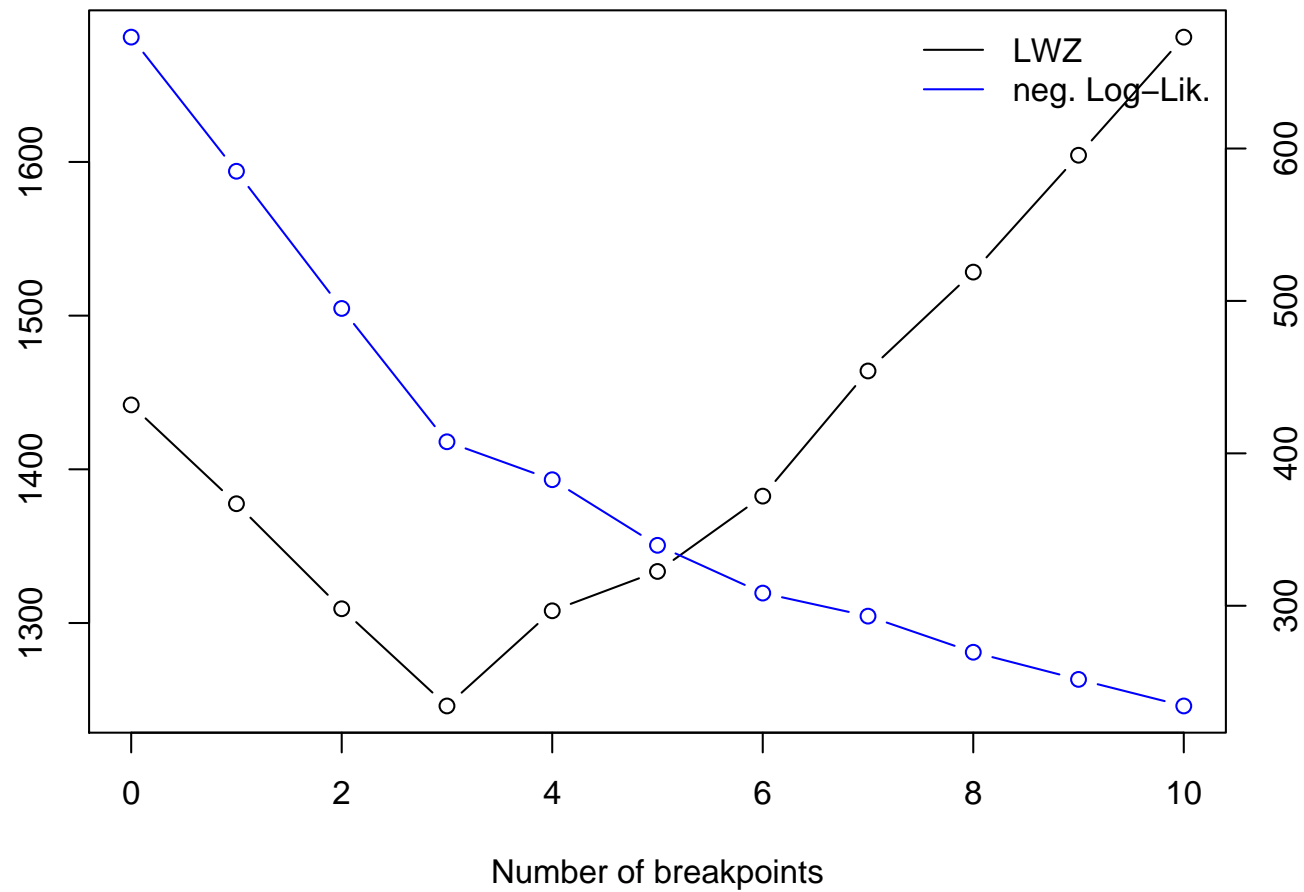
Application: Indian FX regimes

Dating finds the following FX regimes:

start/end	β_0	β_{USD}	β_{JPY}	β_{DUR}	β_{GBP}	σ	R^2
1993-04-09	-0.006	0.972	0.023	0.011	0.020	0.157	0.989
1995-03-03	(0.017)	(0.018)	(0.014)	(0.032)	(0.024)		
1995-03-10	0.161	0.943	0.067	-0.026	0.042	0.924	0.729
1998-08-21	(0.071)	(0.074)	(0.048)	(0.155)	(0.080)		
1998-08-28	0.019	0.993	0.010	0.098	-0.003	0.275	0.969
2004-03-19	(0.016)	(0.016)	(0.010)	(0.034)	(0.021)		
2004-03-26	-0.052	0.748	0.115	0.458	0.133	0.576	0.802
2007-12-28	(0.042)	(0.044)	(0.043)	(0.116)	(0.056)		

1. tight USD peg,
2. flexible USD peg,
3. tight USD peg,
4. flexible basket peg.

Application: Indian FX regimes



Software

All methods are implemented in the R system for statistical computing and graphics

<http://www.R-project.org/>

in the contributed packages **strucchange** available from CRAN and **fxregime** which is under development at R-Forge.

<http://CRAN.R-project.org/>

<http://R-Forge.R-project.org/>

Summary

- Stability of M-type estimator can be assessed by partial sums of the empirical estimating functions.
- FCLT is the basis for inference.
- Test statistics aggregate the empirical fluctuation process to a scalar statistic.
- Significance tests can be enhanced by graphics that bring out timing and/or component of the structural change.
- Framework includes representatives from all important classes of structural change tests based on F statistics, ML scores, OLS residuals.
- The same processes can be used for sequential monitoring.

Summary

- Based on the corresponding objective function the model can also be optimally segmented.
- Software is freely available, works out of the box for linear regression, can be adapted to more general models.
- Structural change tools can be adapted to exchange rate regression by using an approximately normal model.
- Analysis of de facto exchange rate regimes is complemented by methods for testing, monitoring and dating.
- High-level software for analyzing FX regressions is freely available.

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