TECHNISCHE UNIVERSITÄT WIEN
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Monitoring Structural Change in Dynamic Econometric Models

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Contents

Model frame

Consider the linear regression model in a monitoring situation

$$y_i = x_i^\top \beta_i + u_i \qquad (i = 1, \ldots, n, \ldots).$$

Technical assumptions:

- $\limsup_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} ||x_i||^{2+\delta} < \infty$, for some $\delta > 0$.
- $\frac{1}{n} \sum_{i=1}^{n} x_i x_i^\top \xrightarrow{\text{p}} Q$; Q finite, regular, nonstochastic.
- $\{u_i\}$ is a homoskedastic martingale difference sequence.

Model frame

Basic assumption:

The regression relationship is stable ($\beta_i = \beta_0$) during the history period $i = 1, \dots, n$.

Null hypothesis:

$$H_0: \quad \beta_i = \beta_0 \qquad (i > n).$$

Alternative:

$$H_1: \quad \beta_i \neq \beta_0 \qquad \text{for some } i > n.$$

Generalized fluctuation tests

The generalized fluctuation test framework ...

"... includes formal significance tests but its philosophy is basically that of data analysis as expounded by Tukey. Essentially, the techniques are designed to **bring out departures from constancy in a graphic way** *instead of parametrizing particular types of departure in advance and then developing formal significance tests intended to have high power against these particular alternatives."* (Brown, Durbin, Evans, 1975)

Generalized fluctuation tests

- empirical fluctuation processes reflect fluctuation in
  - residuals
  - coefficient estimates

- theoretical limiting process is known

- choose boundaries which are crossed by the limiting process only with a known probability α.

- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large $\Rightarrow$ reject the null hypothesis.

Generalized fluctuation tests

- Chu, Stinchcombe, White (1996)

  Extension of fluctuation tests to the monitoring situation: processes based on recursive estimates and recursive residuals.

- Leisch, Hornik, Kuan (2000)

  Generalized framework for estimates-based tests for monitoring.
  Contains the test of Chu et al., and considered in particular moving estimates.

Generalized fluctuation tests

Processes based on estimates:

$$\hat{\beta}^{(i)} \quad = \quad \left(X_{(i)}{}^{\top} X_{(i)}\right)^{-1} X_{(i)}{}^{\top} y^{(i)}$$

Recursive estimates (RE) process:

$$Y_n(t) = \frac{i}{\hat{\sigma}\sqrt{n}} Q_{(n)}^{\frac{1}{2}} \left(\hat{\beta}^{(i)} - \hat{\beta}^{(n)}\right),$$

where $i = \lfloor k + t(n-k) \rfloor$ and $t \geq 0$.

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where $i = \lfloor k + t(n-k) \rfloor$ and $t \geq 0$.

Moving estimates (ME) process:

$$Z_n(t|\, h) = \frac{\lfloor nh \rfloor}{\hat{\sigma}\sqrt{n}} Q_{(n)}^{\frac{1}{2}} \left(\hat{\beta}^{(\lfloor nt \rfloor - \lfloor nh \rfloor, \lfloor nh \rfloor)} - \hat{\beta}^{(n)}\right),$$

where $t \geq h$.

Generalized fluctuation tests

Limiting processes: (increments of a) k-dimensional Brownian bridge.

Boundaries:

$$\begin{aligned}
\text{RE:} \qquad b(t) &= \sqrt{t(t-1)\left[\lambda^2 + \log\left(\frac{t}{t-1}\right)\right]} \\
\text{ME:} \qquad c(t) &= \lambda \cdot \sqrt{\log_+ t}
\end{aligned}$$

Check for crossings in the monitoring period $1 < t < T$. Significance level is determined by λ.

OLS-based processes

Processes based on OLS residuals:

$$\hat{u}_i \quad = \quad y_i - x_i^\top \hat{\beta}^{(n)}$$

OLS-based CUSUM process:

$$W_n^0(t) \quad = \quad \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \qquad (t \geq 0).$$

OLS-based processes

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$$\hat{u}_i \quad = \quad y_i - x_i^\top \hat{\beta}^{(n)}$$

OLS-based CUSUM process:

$$W_n^0(t) \quad = \quad \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \qquad (t \ge 0).$$

OLS-based MOSUM process:

$$M_n^0(t|h) \quad = \quad \frac{1}{\hat{\sigma}\sqrt{n}} \left(\sum_{i=\lfloor \eta t \rfloor - \lfloor nh \rfloor + 1}^{\lfloor \eta t \rfloor} \hat{u}_i \right) \qquad (t \ge h).$$

OLS-based processes

Limiting processes: (increments of a) 1-dimensional Brownian bridge.

$\Rightarrow$ the same boundaries can be used.

Advantages:

- easy to interpret,
- easy to compute.

Rescaling

Kuan & Chen (1994):

Empirical size of (historical) estimates-based tests can be seriously distorted in dynamic models if the whole inverse sample covariance matrix estimate

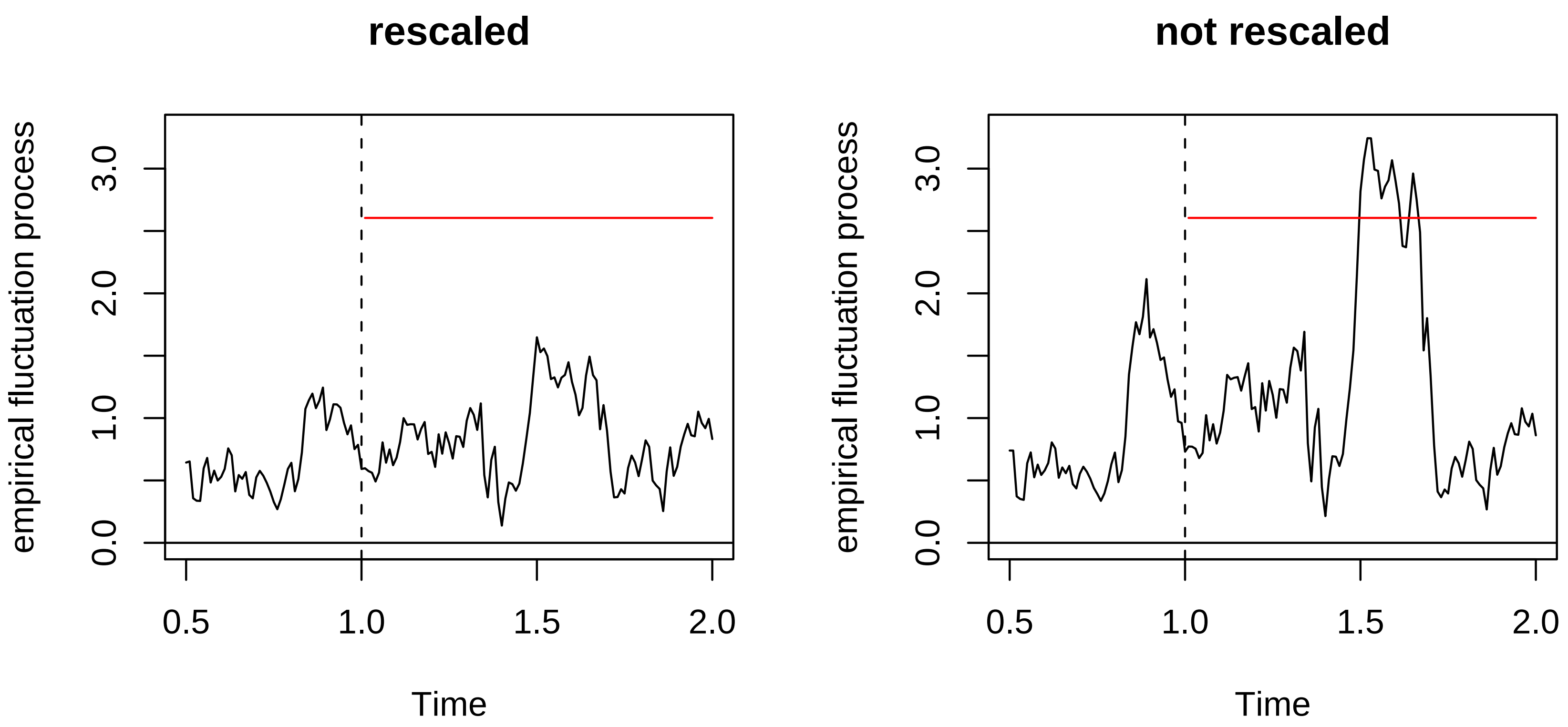
$$Q_{(n)} = 1/n \cdot X_{(n)}^\top X_{(n)}$$

is used to scale the fluctuation process.
Improvement: use $Q_{(i)}$ instead.

In a monitoring situation rescaling cannot improve the size of the RE test but it does so for the ME test!

Rescaling

Example: AR(1) process with $\varrho = 0.9$ but **without** a shift:



German M1 money demand

Lütkepohl, Teräsvirta, Wolters (1999) investigate the linearity and stability of German M1 money demand: stable regression relation for the time before the monetary unification on 1990-06-01 but a clear structural instability afterwards.

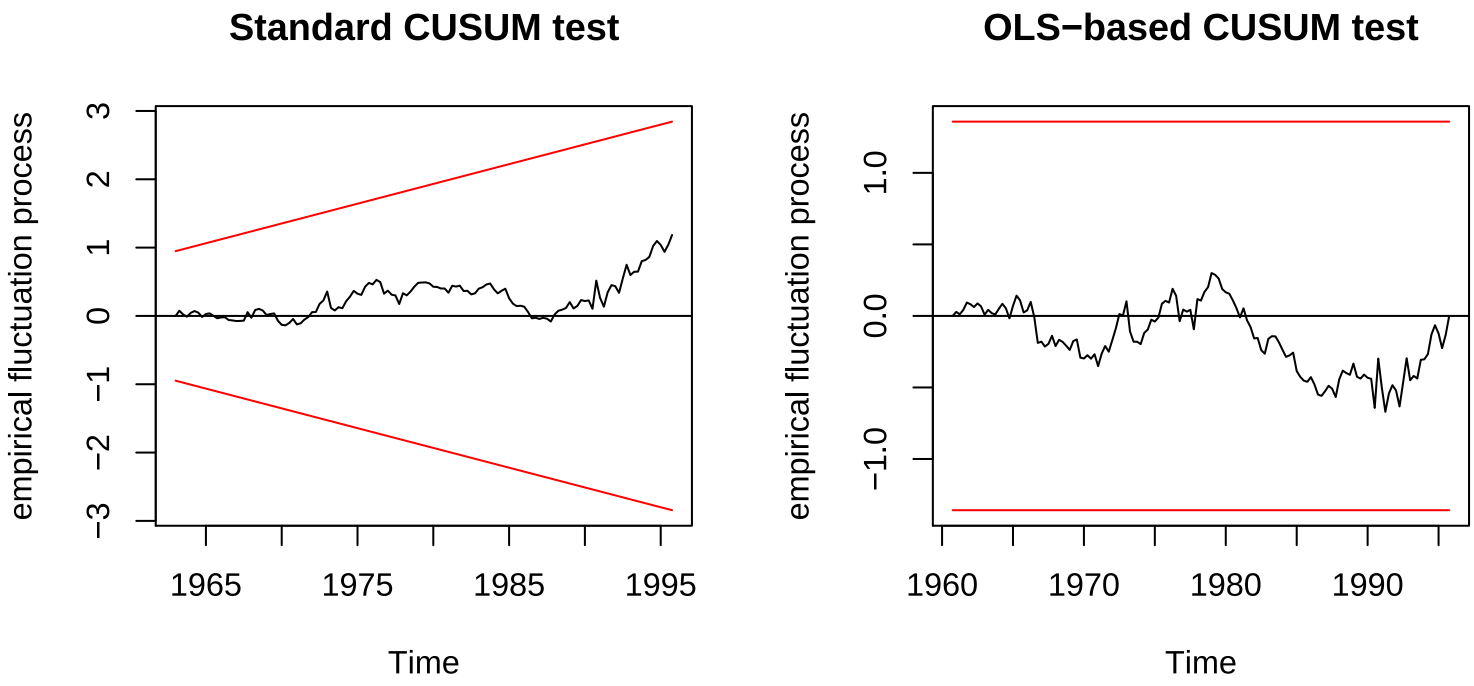
Data: seasonally unadjusted quarterly data, 1961(1) to 1995(4)

Error Correction Model (in logs) with variables:
M1 (real, per capita) m_t, price index p_t, GNP (real, per capita) y_t and long-run interest rate R_t:

$$\begin{aligned}\Delta m_t \;=\; & -0.30\Delta y_{t-2} - 0.67\Delta R_t - 1.00\Delta R_{t-1} - 0.53\Delta p_t \\ & -0.12 m_{t-1} + 0.13 y_{t-1} - 0.62 R_{t-1} \\ & -0.05 - 0.13 Q1 - 0.016 Q2 - 0.11 Q3 + \hat{u}_t,\end{aligned}$$

German M1 money demand

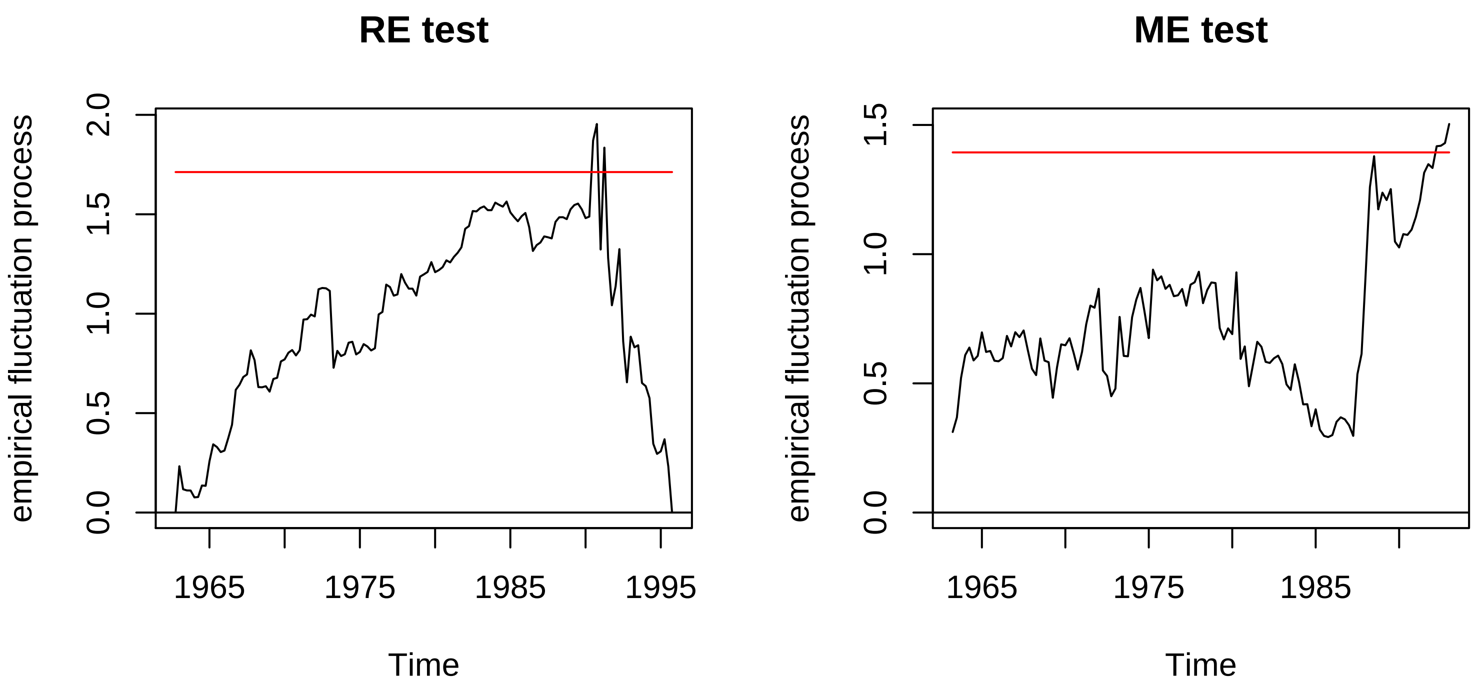
Historical residual-based tests...do **not** discover shift:



The shift has an estimated angle of 90.11°.

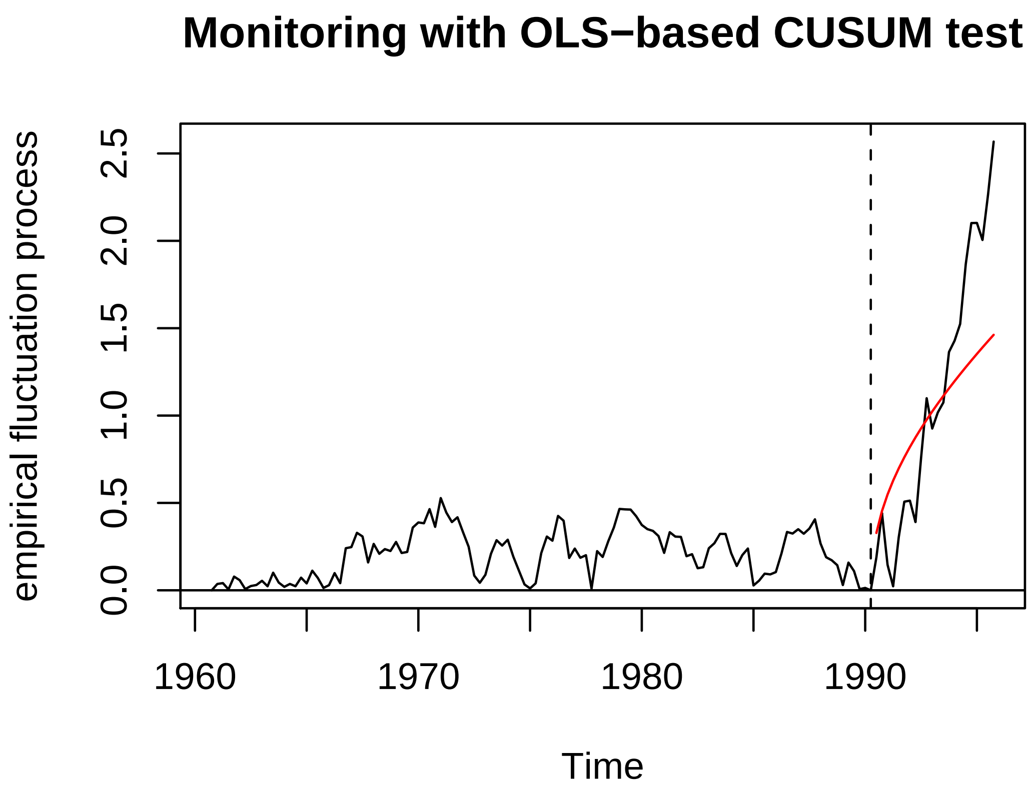
German M1 money demand

Historical estimates-based tests discover shift **ex post**:



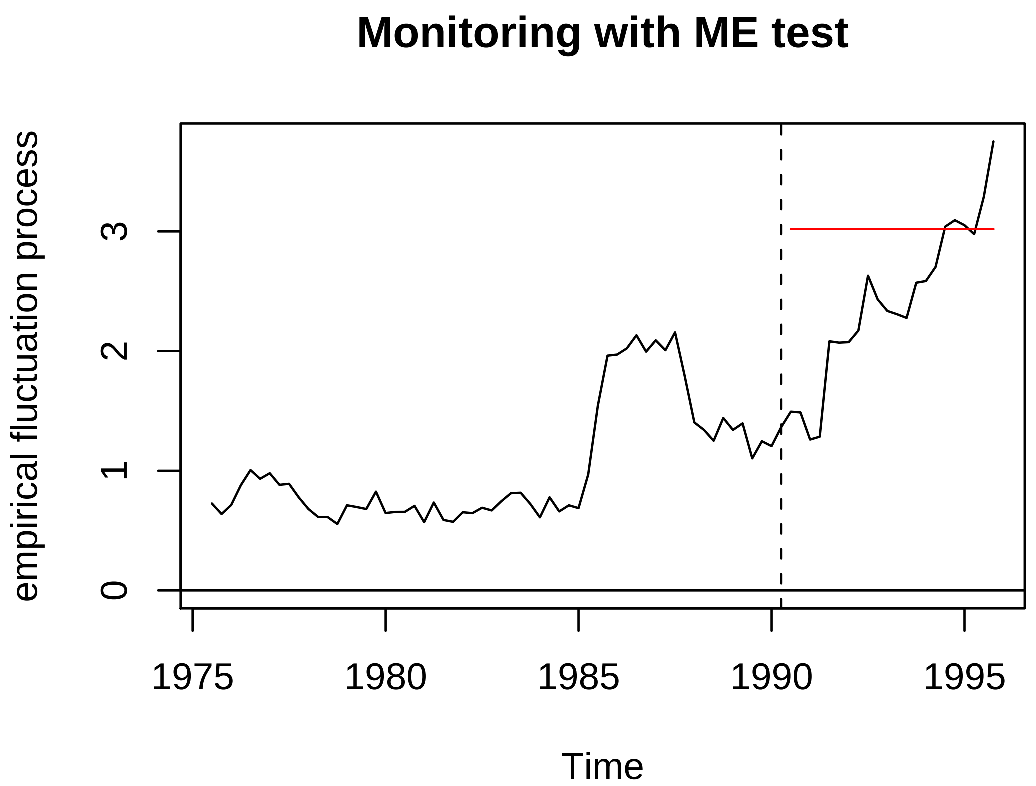
German M1 money demand

Monitoring discovers shift **online**:



German M1 money demand

Monitoring discovers shift **online**:



Software

All methods implemented in the R system for statistical computing and graphics

`http://www.R-project.org/`

in the contributed package `strucchange`.

Both are available under the GPL (General Public Licence) from the Comprehensive R Archive Network (CRAN):

`http://cran.R-project.org/`

Software

Further functionality:

- Historical tests:
  - Generalized fluctuation tests (Kuan & Hornik): CUSUM, MOSUM, RE, ME, Nyblom-Hansen,
  - F tests (Andrews & Ploberger): sup$F$, ave$F$, exp$F$.
- Dating breaks (Bai & Perron):
  - breakpoint estimation,
  - confidence intervals.

Software

Documented in:

A. Zeileis, F. Leisch, K. Hornik, C. Kleiber (2002), "`strucchange`: An R Package for Testing for Structural Change in Linear Regression Models," *Journal of Statistical Software*, 7(2), 1–38.

A. Zeileis, C. Kleiber, W. Krämer, K. Hornik (2003), "Testing and Dating Structural Changes in Practice," *Computational Statistics & Data Analysis*, forthcoming.

Software

```
R> LTW.model <- dm ~ dy2 + dR + dR1 + dp + m1 + y1 + R1 + season
R> re <- efp(LTW.model, type = "RE", data = GermanM1)
R> plot(re)
```

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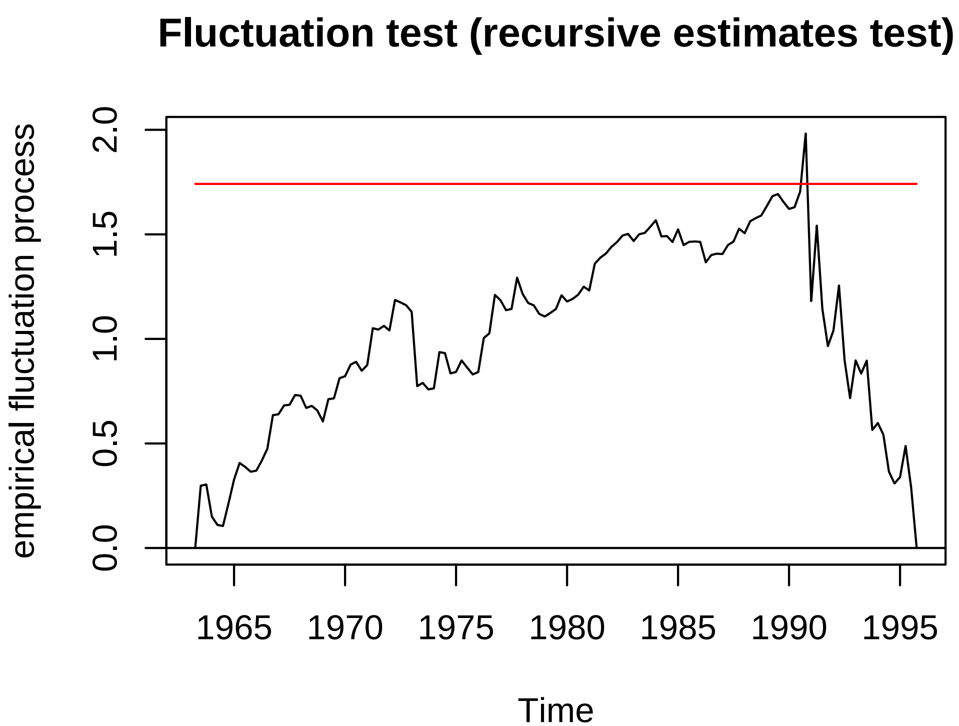

Software

```
R> sctest(re)

        Fluctuation test (recursive estimates test)

data:  re
FL = 1.9821, p-value = 0.008475
```