Monitoring Structural Change in Dynamic Econometric Models

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Consider the linear regression model in a monitoring situation

\[ y_i = x_i \beta_i + u_i \quad (i = 1, \ldots, n, \ldots). \]

Technical assumptions:

- \( \limsup_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \|x_i\|^{2+\delta} < \infty, \) for some \( \delta > 0. \)

- \( \frac{1}{n} \sum_{i=1}^{n} x_i x_i^\top \xrightarrow{p} Q; \) \( Q \) finite, regular, nonstochastic.

- \( \{u_i\} \) is a homoskedastic martingale difference sequence.
Basic assumption:

The regression relationship is stable ($\beta_i = \beta_0$) during the history period $i = 1, \ldots, n$.

Null hypothesis:

$$H_0 : \quad \beta_i = \beta_0 \quad (i > n).$$

Alternative:

$$H_1 : \quad \beta_i \neq \beta_0 \quad \text{for some } i > n.$$
The generalized fluctuation test framework ...

“... includes formal significance tests but its philosophy is basically that of data analysis as expounded by Tukey. Essentially, the techniques are designed to bring out departures from constancy in a graphic way instead of parametrizing particular types of departure in advance and then developing formal significance tests intended to have high power against these particular alternatives.” (Brown, Durbin, Evans, 1975)
Generalized fluctuation tests

- empirical fluctuation processes reflect fluctuation in
  - residuals
  - coefficient estimates

- theoretical limiting process is known

- choose boundaries which are crossed by the limiting process only with a known probability $\alpha$.

- if the empirical fluctuation process crosses the theoretical boundaries the fluctuation is improbably large $\Rightarrow$ reject the null hypothesis.
Generalized fluctuation tests

- Chu, Stinchcombe, White (1996)
  Extension of fluctuation tests to the monitoring situation: processes based on recursive estimates and recursive residuals.

- Leisch, Hornik, Kuan (2000)
  Generalized framework for estimates-based tests for monitoring.
  Contains the test of Chu et al., and considered in particular moving estimates.
Generalized fluctuation tests

Processes based on estimates:

\[ \hat{\beta}^{(i)} = \left( X_{(i)}^\top X_{(i)} \right)^{-1} X_{(i)}^\top y^{(i)} \]

Recursive estimates (RE) process:

\[ Y_n(t) = \frac{i}{\hat{\sigma} \sqrt{n}} Q_{(n)}^{1/2} \left( \hat{\beta}^{(i)} - \hat{\beta}^{(n)} \right), \]

where \( i = [k + t(n - k)] \) and \( t \geq 0 \).
Generalized fluctuation tests

Processes based on estimates:

$$\hat{\beta}(i) = \left( X(i)^\top X(i) \right)^{-1} X(i)^\top y(i)$$

Recursive estimates (RE) process:

$$Y_n(t) = i \hat{\sigma} \sqrt{n} Q_{(n)}^{1/2} \left( \hat{\beta}(i) - \hat{\beta}(n) \right),$$

where $i = \lfloor k + t(n - k) \rfloor$ and $t \geq 0$.

Moving estimates (ME) process:

$$Z_n(t|h) = \left\lfloor nh \right\rfloor \hat{\sigma} \sqrt{n} Q_{(n)}^{1/2} \left( \hat{\beta}\left(\left\lfloor nt \right\rfloor - \left\lfloor nh \right\rfloor, \left\lfloor nh \right\rfloor \right) - \hat{\beta}(n) \right),$$

where $t \geq h$. 
Generalized fluctuation tests

The empirical processes converge to a $k$-dimensional Brownian bridge or the increments thereof respectively.

The null hypothesis is rejected when the empirical processes cross the boundary

\[ b(t) = \sqrt{t(t-1)} \left[ \lambda^2 + \log \left( \frac{t}{t-1} \right) \right] \]

or

\[ c(t) = \lambda \cdot \sqrt{\log_+ t} \]

respectively in the monitoring period $1 < t < T$ and $\lambda$ determines the significance level of this procedure.
Processes based on OLS residuals:

\[ \hat{u}_i = y_i - x_i^\top \hat{\beta}(n) \]

OLS-based CUSUM process:

\[ W_{n}^{0}(t) = \frac{1}{\hat{\sigma}/\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \quad (t \geq 0). \]
OLS-based processes

Processes based on OLS residuals:

\[
\hat{u}_i = y_i - x_i^\top \hat{\beta}(n)
\]

OLS-based CUSUM process:

\[
W_n^0(t) = \frac{1}{\hat{\sigma} \sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \quad (t \geq 0).
\]

OLS-based MOSUM process:

\[
M_n^0(t|h) = \frac{1}{\hat{\sigma} \sqrt{n}} \left( \sum_{i=\lfloor \eta t \rfloor - \lfloor nh \rfloor + 1}^{\lfloor \eta t \rfloor} \hat{u}_i \right) \quad (t \geq h).
\]
The limiting processes are the 1-dimensional Brownian bridge or the increments thereof respectively. Thus, the same boundaries can be used.

Advantage: ease of computation.
Kuan & Chen (1994):

Empirical size of (historical) estimates-based tests can be seriously distorted in dynamic models if the whole sample covariance matrix estimate

\[ Q(n) = \frac{1}{n} \cdot X_{(n)}^\top X_{(n)} \]

is used to scale the fluctuation process.

Improvement: use \( Q(i) \) instead.

In a monitoring situation rescaling cannot improve the size of the RE test but it does so for the ME test!
Example: AR(1) process with $\varrho = 0.9$ but \textit{without} a shift:
Lütkepohl, Teräsvirta, Wolters (1999) investigate the linearity and stability of German M1 money demand: stable regression relation for the time before the monetary union on 1990-06-01 but a clear structural instability afterwards.

Data: seasonally unadjusted quarterly data, 1961(1) to 1995(4)

**Error Correction Model** (in logs) with variables:
M1 (real, per capita) $m_t$, price index $p_t$, GNP (real, per capita) $y_t$ and long-run interest rate $R_t$:

$$
\Delta m_t = -0.30 \Delta y_{t-2} - 0.67 \Delta R_t - 1.00 \Delta R_{t-1} - 0.53 \Delta p_t \\
-0.12 m_{t-1} + 0.13 y_{t-1} - 0.62 R_{t-1} \\
-0.05 - 0.13 Q1 - 0.016 Q2 - 0.11 Q3 + \tilde{u}_t,
$$
Historical residual-based tests...do not discover shift:

The shift has an estimated angle of 90.27°.
German M1 money demand

Historical estimates-based tests discover shift *ex post*:

RE test

ME test
Monitoring discovers shift *online*:
Monitoring discovers shift *online*:
All methods implemented in R

http://www.R-project.org/

in the contributed package strucchange available from the Comprehensive R Archive Network (CRAN):

http://cran.R-project.org/

documented in:
R> LTW.model <- dm ~ dy2 + dR + dR1 + dp + m1 + y1 + R1 + season
R> re <- efp(LTW.model, type = "RE", data = GermanM1)
R> plot(re)
R> LTW.model <- dm ~ dy2 + dR + dR1 + dp + m1 + y1 + R1 + season
R> re <- efp(LTW.model, type = "RE", data = GermanM1)
R> plot(re)
R> sctest(re)

Fluctuation test (recursive estimates test)

data:  re
FL = 1.9821, p-value = 0.008475