



# **Automatic Generation of Simple (Statistical) Exams**

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# Overview

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# Introduction

## Re-design of introductory statistics lecture at WU Wien:

- The course is attended each semester by 1,000–1,500 students (mostly first-year business students).
- Several lecturers from the Department of Statistics and Mathematics teach this course in parallel.
- All teaching materials are covered by the re-design: presentation slides, collections of exercises, exams, etc.
- The re-design was accomplished through a collaborative effort of all concerned faculty members working in small teams on different chapters.

# Introduction / 2

## Main challenges:

- *Scalable exams*: Automatic generation of a large number of different exams.
- *Associated self-study materials*: Collections of exercises and solutions from the same pool of examples.
- *Joint development*: Development and maintenance of a large pool of exercises in a multi-author and cross-platform setting.

## Tools chosen:

- R (R Development Core Team 2008) and  $\text{\LaTeX}$  (Knuth 1984; Lamport 1994)  $\Rightarrow$  Sweave (Leisch 2002)
- Subversion (SVN, Pilato, Collins-Sussman, and Fitzpatrick 2004)

# Introduction / 3

## Design principles of package exams:

- *Maintenance*: Each exercise template is a single file (also just called “exercise”).
- *Variation*: Exercises are dynamic documents, containing a problem/solution along with a data-generating process (DGP) so that random samples can be drawn easily.
- *Correction*: Solutions for exercises are either multiple-choice answers (logical vectors) or numeric values (e.g., a test statistic or a confidence interval).

# Exercises

Each exercise typically represents an exemplary application of a statistical procedure.

The exercise file consists of (at least):

- **Two environments:** a question and a solution description encapsulated in corresponding  $\text{\LaTeX}$  environments.
- **Meta-information:** about type of questions (e.g. multiple-choice or numeric), the solution, a descriptive name and the allowed tolerance for numeric solutions.

An exercise file can be processed in R by:

```
R> library("exams")
R> tstat_sol <- exams("tstat.Rnw")
R> tstat_sol

plain1
  1. t statistic: 15.958 (15.948--15.968)
```

# A simple Sweave exercise: tstat.Rnw

```
<<echo=FALSE, results=hide>>=
## DATA GENERATION
n <- sample(120:250, 1)
mu <- sample(c(125, 200, 250, 500, 1000), 1)
y <- rnorm(n, mean = mu * runif(1, min = 0.9, max = 1.1),
           sd = mu * runif(1, min = 0.02, max = 0.06))
## QUESTION/ANSWER GENERATION
Mean <- round(mean(y), digits = 1)
Var <- round(var(y), digits = 2)
tstat <- round((Mean - mu)/sqrt(Var/n), digits = 3)
@
\begin{question}
  A machine fills milk into  $\$ \backslash \text{Sexpr}\{\mu\} \$ \text{ml}$  packages. It is suspected that the
  ...
\end{question}
\begin{solution}
  ...
\end{solution}

%% META-INFORMATION
%% \extype{num}
%% \exsolution{\Sexpr{format(abs(tstat), nsmall = 3)}}
%% \exname{t statistic}
%% \extol{0.01}
```

# L<sup>A</sup>T<sub>E</sub>X output of Sweave ("tstat.Rnw")

```
\begin{question}
```

A machine fills milk into \$500\$ml packages. It is suspected that the machine is not working correctly and that the amount of milk filled differs from the setpoint  $\mu_0 = 500$ . A sample of \$226\$ packages filled by the machine are collected. The sample mean  $\bar{y}$  is equal to \$517.2\$ and the sample variance  $s^2_{n-1}$  is equal to \$262.56\$.

Test the hypothesis that the amount filled corresponds on average to the setpoint. What is the absolute value of the  $t$ -test statistic?

```
\end{question}
```

```
\begin{solution}
```

The  $t$ -test statistic is calculated by:

```
\begin{eqnarray*}
```

$$t = \frac{\bar{y} - \mu_0}{\sqrt{\frac{s^2_{n-1}}{n}}} = \frac{517.2 - 500}{\sqrt{\frac{262.56}{226}}} = 15.958.$$

```
\end{eqnarray*}
```

The absolute value of the  $t$ -test statistic is thus equal to \$15.958\$.

```
\end{solution}
```

```
%% META-INFORMATION
```

```
%% \extype{num}
```

```
%% \exsolution{15.958}
```

```
%% \exname{t statistic}
```

```
%% \extol{0.01}
```



# Display of processed tstat exercise

- **Problem**

A machine fills milk into 500ml packages. It is suspected that the machine is not working correctly and that the amount of milk filled differs from the setpoint  $\mu_0 = 500$ . A sample of 226 packages filled by the machine are collected. The sample mean  $\bar{y}$  is equal to 517.2 and the sample variance  $s_{n-1}^2$  is equal to 262.56.

Test the hypothesis that the amount filled corresponds on average to the setpoint. What is the absolute value of the  $t$  test statistic?

**Solution**

The  $t$  test statistic is calculated by:

$$t = \frac{\bar{y} - \mu_0}{\sqrt{\frac{s_{n-1}^2}{n}}} = \frac{517.2 - 500}{\sqrt{\frac{262.56}{226}}} = 15.958.$$

The absolute value of the  $t$  test statistic is thus equal to 15.958.

# Combining exercises: The master $\text{\LaTeX}$ file

`exams()` allows for

- construction of exams with stratified sampling of exercises,
- automatic generation of multiple copies (potentially of multiple layouts) with suitable names and storage,
- inclusion of a suitable cover page with answer fields, and
- collection of meta-information for problems and solutions in an R object.

## Sequence of work steps for `exams()`

- 1 Collect all Sweave files for the exercises, the master  $\text{\LaTeX}$  file(s) and potentially additionally specified input files.
- 2 Copy all files to a (temporary, by default) directory.
- 3 Run `Sweave()` for each exercise.
- 4 Produce a copy of the master  $\text{\LaTeX}$  file(s) in which certain control structures are substituted by dynamically generated  $\text{\LaTeX}$  commands (e.g., for including the exercises).
- 5 Run `texi2dvi()` for each master  $\text{\LaTeX}$  file.
- 6 Store the resulting PDF file(s) in an output directory or display it on the screen (for a single file only, by default).

# A simple master $\text{\LaTeX}$ file: plain.tex

```
\documentclass[a4paper]{article}

\usepackage{a4wide,Sweave}
\newenvironment{question}{\item \textbf{Problem}\newline}{ }
\newenvironment{solution}{\textbf{Solution}\newline}{ }

\begin{document}
\begin{enumerate}
%% \exinput{exercises}
\end{enumerate}
\end{document}
```

To hide the solution the corresponding environment needs to be defined as a comment:

```
\newenvironment{solution}{\comment}{\endcomment}}
```

## Possible dynamic modifications

- `\exinput{exercises}`: Inclusion of exercises.  
Replaced by: `\input{filename}` (one for each exercise).  
Example: `\input{tstat}`.
- `\exinput{questionnaire}`: Inclusion of questionnaires, e.g., for cover sheets.  
Replaced by: `\exnum{...}` or `\exmchoice{...}`, respectively (one for each exercise).  
Example: `\exnum{}{}{}{}{1}{5}{9}{5}{8}`.
- `\exinput{header}`: Further commands and definitions.  
Replaced by: `\command{value}` (one for each header command).  
Example: `\Date{2009-01-16}`.

## Application and customization: Function `exams()`

Function `exams()` has the following arguments:

```
exams(file, n = 1, dir = NULL,  
       template = "plain", inputs = NULL,  
       header = list(Date = Sys.Date()),  
       name = NULL, quiet = TRUE, edir = NULL,  
       tdir = NULL, control = NULL)
```

## Application and customization: Function exams() / 2

```
R> myexam <- list("boxplots",
+   c("confint", "ttest", "tstat"),
+   c("anova", "regression"),
+   "scatterplot",
+   "relfreq")
R> odir <- tempfile()
R> getID <- function(i)
+   paste("myexam", gsub(" ", "0", format(i, width = 2)),
+         sep = "")
R> getID(1)

[1] "myexam01"
```

## Application and customization: Function exams() / 3

```
R> set.seed(1090)
R> sol <- exams(myexam, n = 3, dir = odir,
+   template = c("exam", "solution"),
+   header = list(ID = getID, Date = Sys.Date()))
R> list.files(odir)

[1] "exam1.pdf"      "exam2.pdf"      "exam3.pdf"
[4] "metainfo.rda"  "solution1.pdf"  "solution2.pdf"
[7] "solution3.pdf"

R> print(sol, 1)

exam1
  1. Multiple choice: abde
  2. t statistic: 0.188 (0.178--0.198)
  3. Prediction: 236.678 (236.668--236.688)
  4. Multiple choice: abde
  5. Multiple choice: d
```



# Application and customization: Function exams () / 4

R University

Statistics Exam 2009-01-16

Exam ID myexam01

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Signature: \_\_\_\_\_

1. (a)  (b)  (c)  (d)  (e)
2.  :
3.  :
4. (a)  (b)  (c)  (d)  (e)
5. (a)  (b)  (c)  (d)  (e)

Statistics Exam: myexam01

2

1. In Figure 1 the distributions of a variable given by two samples (A and B) are represented by parallel boxplots. Which of the following statements are correct? (Comment: The statements are either about correct or clearly wrong.)

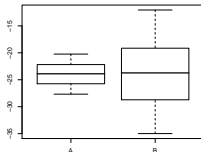


Figure 1: Parallel boxplots.

- (a) The location of both distributions is about the same.  
(b) Both distributions contain no outliers.  
(c) The spread in sample A is clearly bigger than in B.  
(d) The skewness of both samples is similar.  
(e) Distribution A is about symmetric.
2. A machine fills milk into 500ml packages. It is suspected that the machine is not working correctly and that the amount of milk filled differs from the setpoint  $\mu_0 = 500$ . A sample of 225 packages filled by the machine are collected. The sample mean  $\bar{y}$  is equal to 499.7 and the sample variance  $s_y^2$  is equal to 576.1.  
Test the hypothesis that the amount filled corresponds on average to the setpoint. What is the absolute value of the  $t$  test statistic?
3. For 49 firms the number of employees  $X$  and the amount of expenses for continuing education  $Y$  (in EUR) were recorded. The statistical summary of the data set is given by:

	Variable X	Variable Y
Mean	58	232
Variance	124	1606

- The correlation between  $X$  and  $Y$  is equal to 0.65.  
Estimate the expected amount of money spent for continuing education by a firm with 60 employees using least squares regression.
4. Figure 2 shows a scatterplot. Which of the following statements are correct?

## Application and customization: Function exams() / 5

Several arguments allow for a fine control, e.g. to modify the print output:

```
R> mycontrol <- list(mchoice.print =  
+   list(True = LETTERS[1:5], False = "_"))  
R> (exams(myexam, n = 1, template = "exam",  
+   control = mycontrol))
```

exam1

1. Multiple choice: \_B\_D\_
2. Multiple choice: AB\_D\_
3. Prediction: 208.13 (208.12--208.14)
4. Multiple choice: \_\_C\_\_
5. Multiple choice: -----

# Discussion

- Package **exams** provides a framework for automatic generation of simple (statistical) exams and associated self-study materials.
- It is based on independent exercises in Sweave format which can be compiled in exams (or other collections of exercises) by providing one (or more) master  $\text{\LaTeX}$  template(s).
- Contributing to the pool of exercises only requires knowledge of Sweave and minimal markup for meta-information.
- Since Spring 2008, **exams** is used at the WU Wien for generating collections of exercises, trial exams, exams and solutions.

Package **exams** is available from the Comprehensive R Archive Network at

`http://CRAN.R-project.org/package=exams`

# References

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